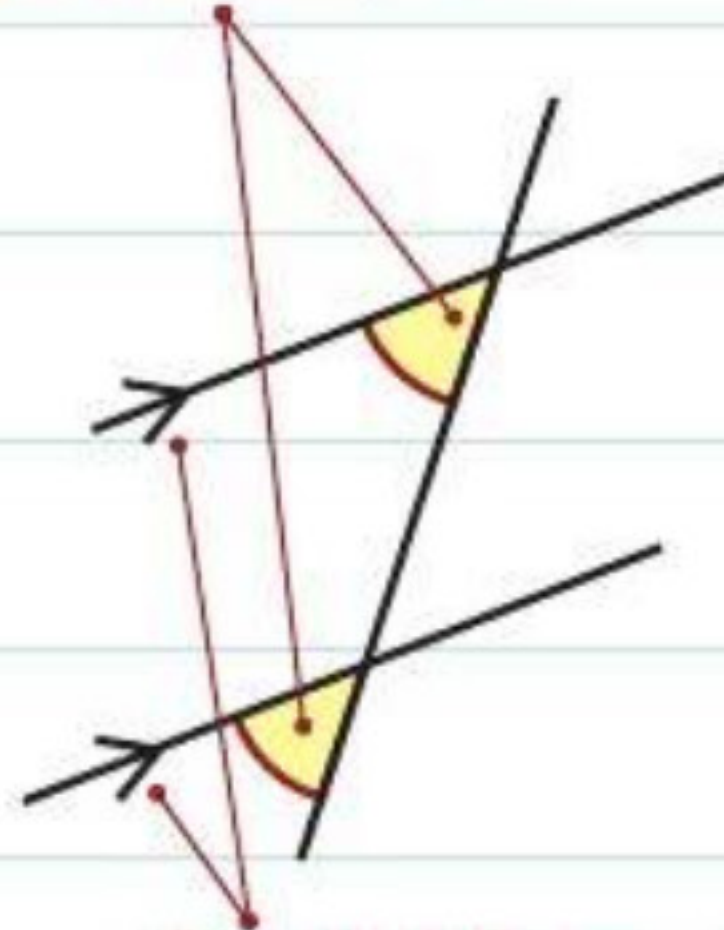# Angle properties
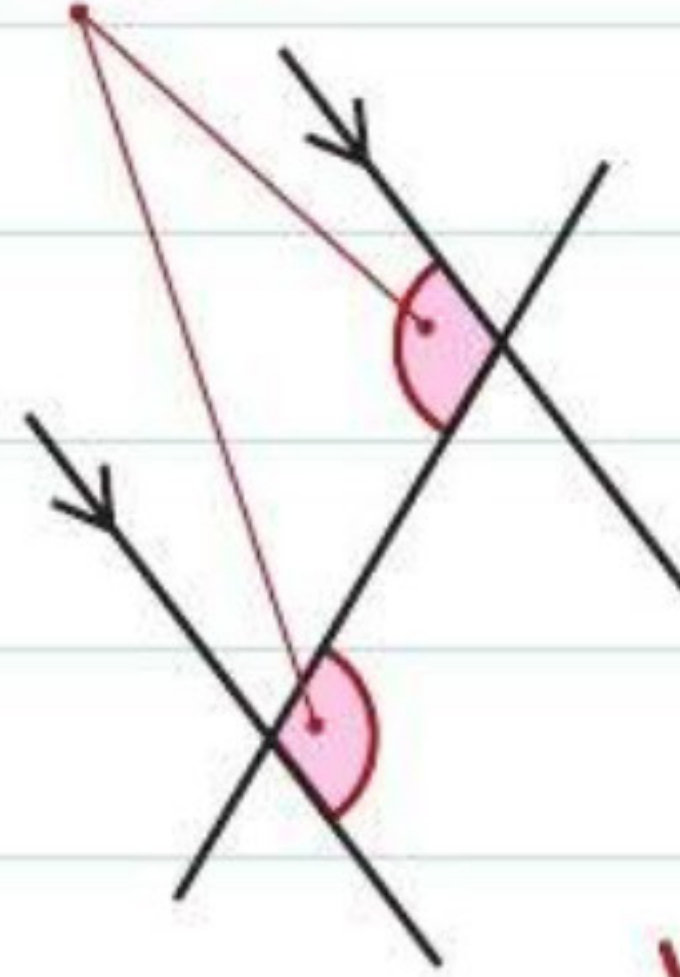
You need to remember all of these angle properties and their correct names.

Corresponding angles are equal.

Alternate angles are equal.
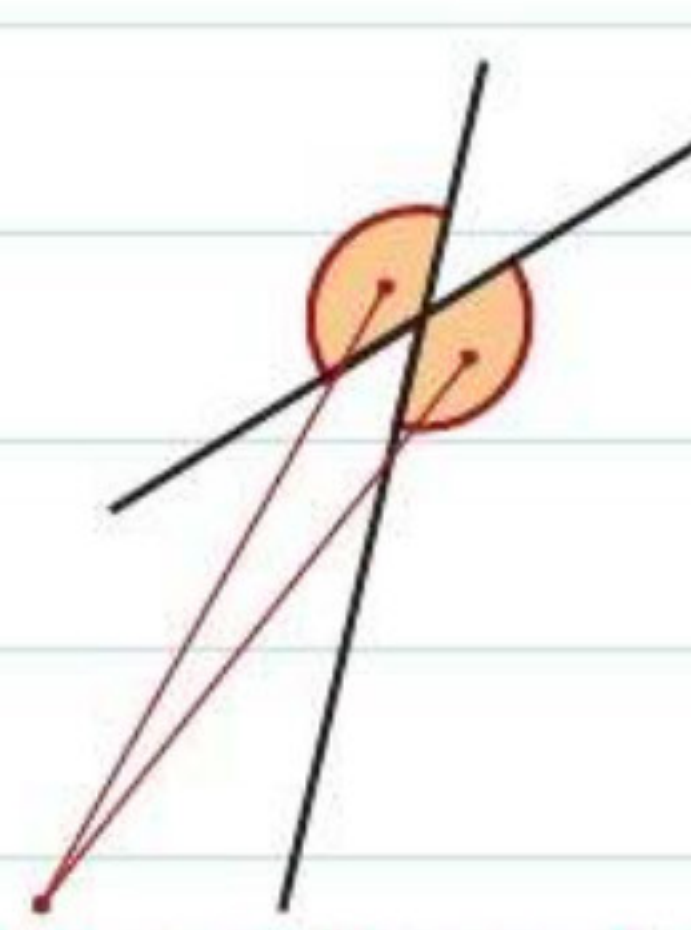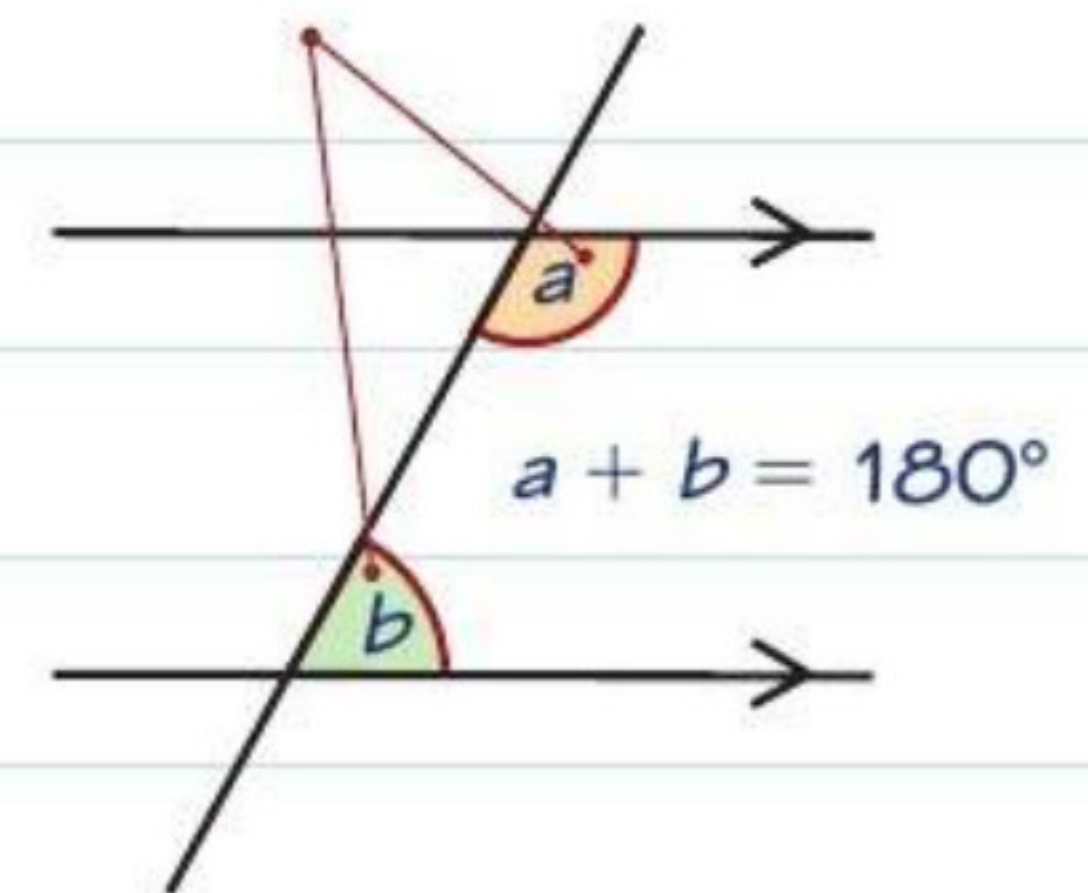
Co-interior or allied angles add up to 180°.

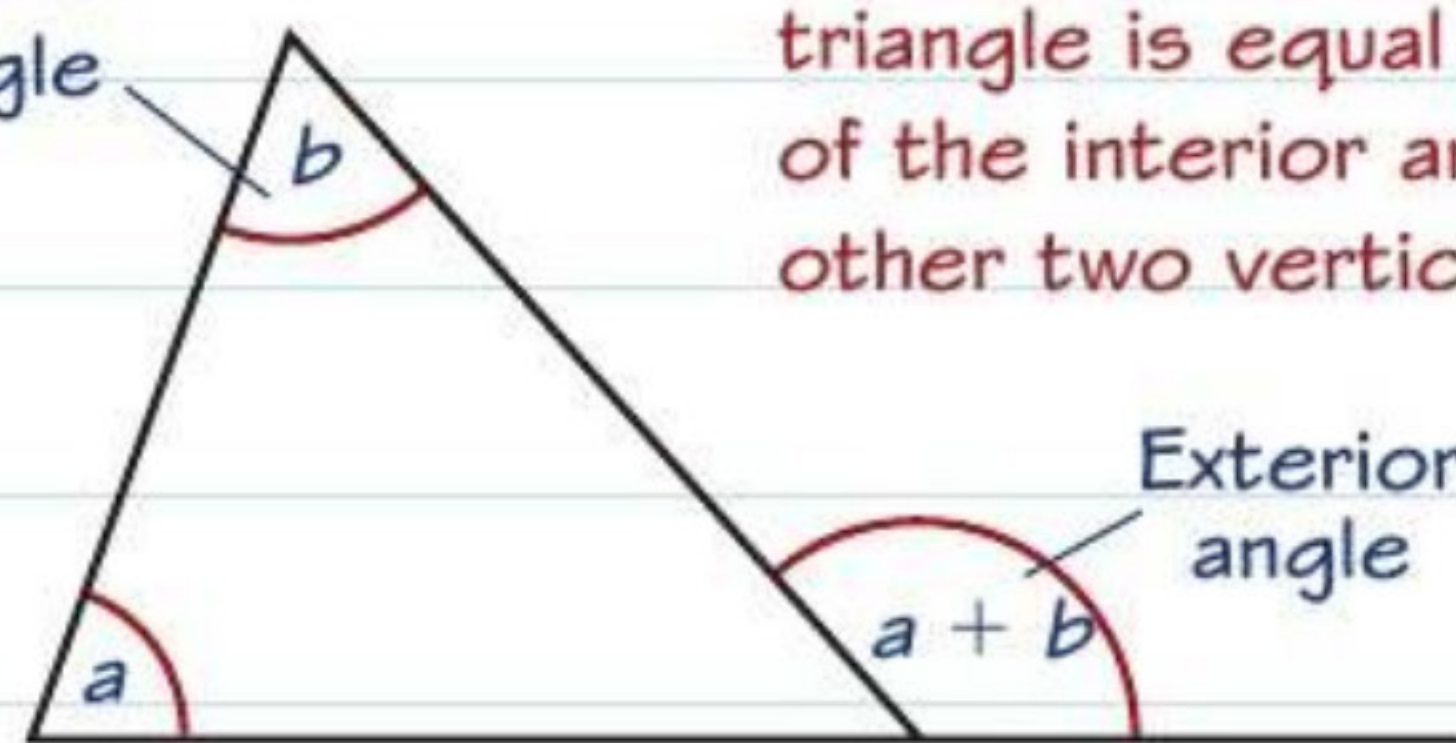$a + b = 180°$

Parallel lines are marked with arrows.
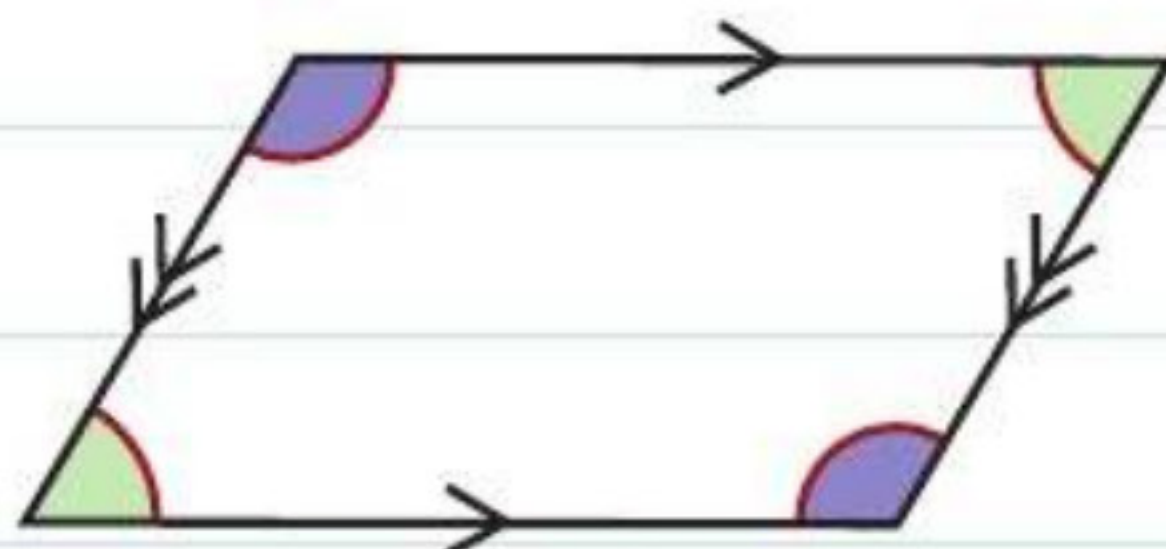
Vertically opposite angles are equal.

These are useful angle facts for triangles and parallelograms:

Interior angle

The exterior angle of a triangle is equal to the sum of the interior angles at the other two vertices.

Exterior angle

$a + b$

The opposite angles of a parallelogram are equal.

You need to know the proofs of the angle properties of triangles and quadrilaterals.

Golden rule

When answering angle problems, you need to give a reason for each step of your working.

Angle sums

You need to remember these two angle facts:

1 The angles in a triangle add up to 180°.

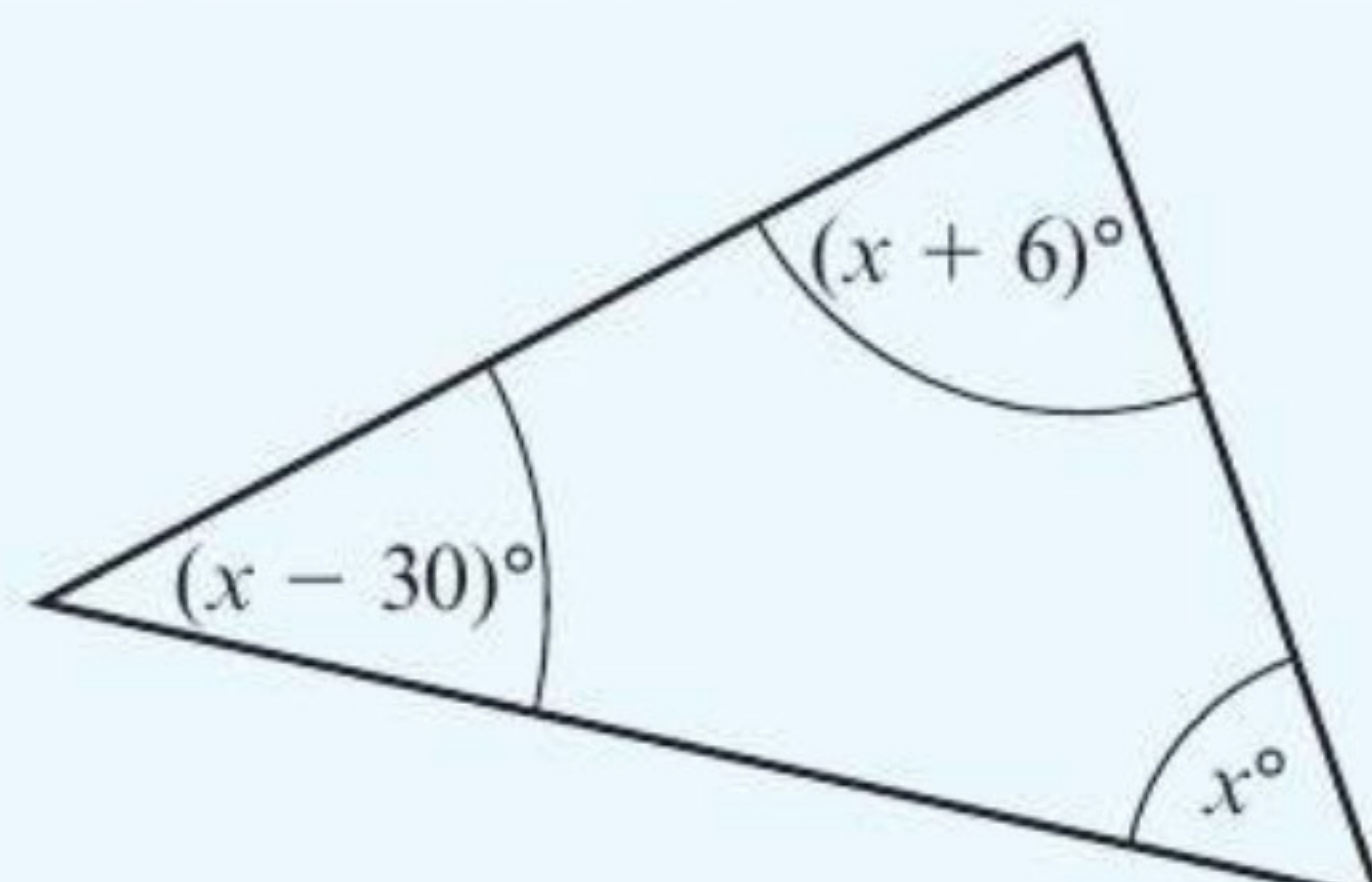**2** The angles in a quadrilateral add up to 360°.

Problem solved! Use the fact that the angles in a triangle add up to 180° to write an equation, then solve your equation to find x. For a reminder about solving linear equations have a look at page 19.

Worked example

Target grade 5

$(x + 6)°$

$(x - 30)°$

$x°$

Work out the value of x.   **(3 marks)**

Angles in a triangle add up to 180° so

$(x - 30) + (x + 6) + x = 180$

$3x - 24 = 180$ $(+ 24)$

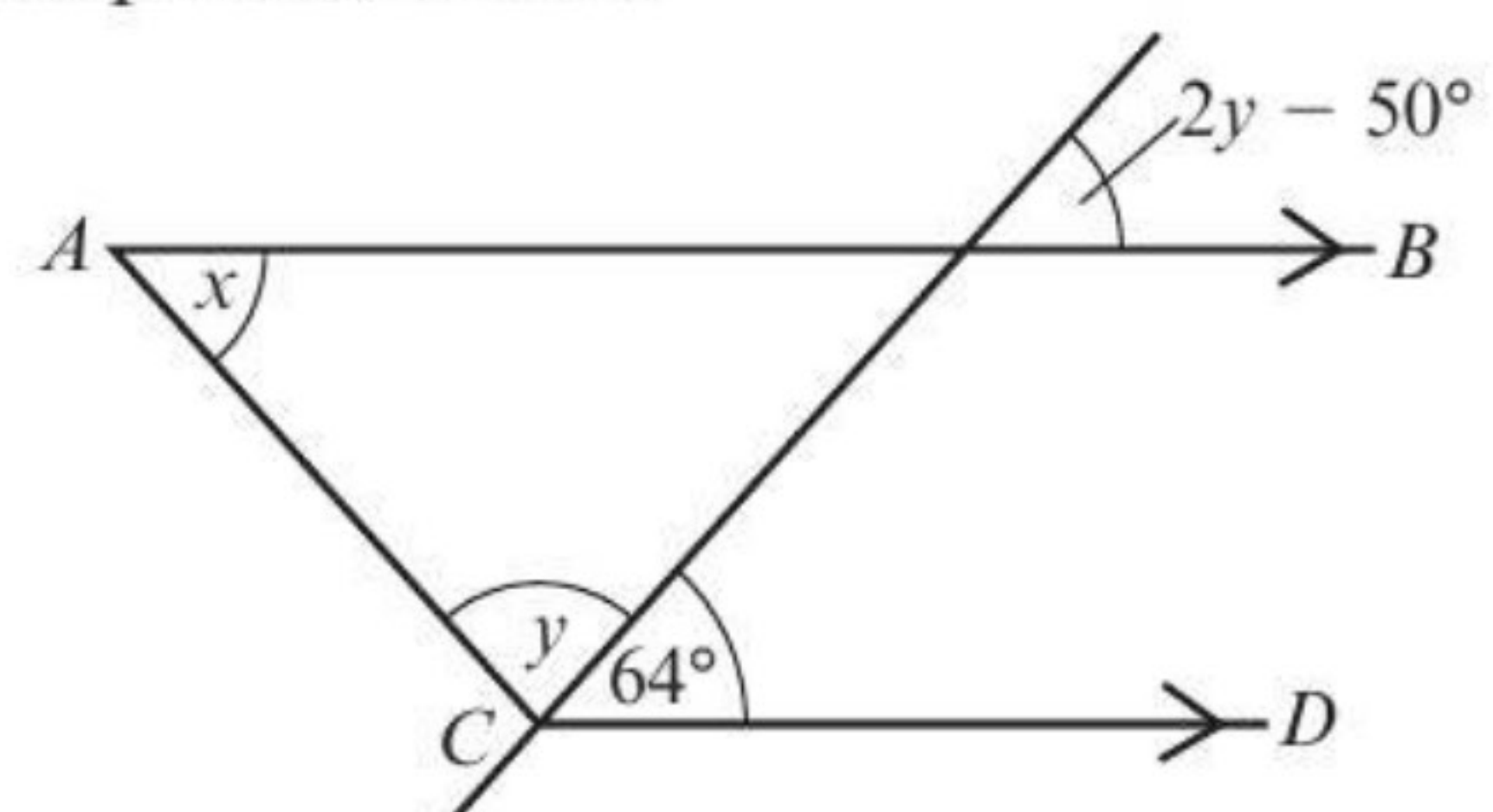$3x = 204$ $(÷ 3)$

$x = 68$

Now try this

Target grade 5

AB is parallel to CD.

A x

$2y - 50°$ B

y $64°$

C D

Work out the value of x.   **(5 marks)**

Hot Topic

Solving angle problems

You might need to use angle properties to solve problems in your exam. Remember to give reasons for every step of your working.

Reasons

Use these reasons in angle problems:
- Angles on a straight line add up to 180°.
- Angles around a point add up to 360°.
- Opposite angles are equal.
- Corresponding angles are equal.
- Co-interior angles add up to 180°.
- Alternate angles are equal.
- Angles in a triangle add up to 180°.
- Angles in a quadrilateral add up to 360°.
- Base angles of an isosceles triangle are equal.

Use the properties on the diagram:
AB is parallel to CD.
AC is parallel to BD.
BE is equal to DE.

Worked example

Target grade **5**

Work out the size of the angle marked x. Give reasons for each step of your working.

(4 marks)
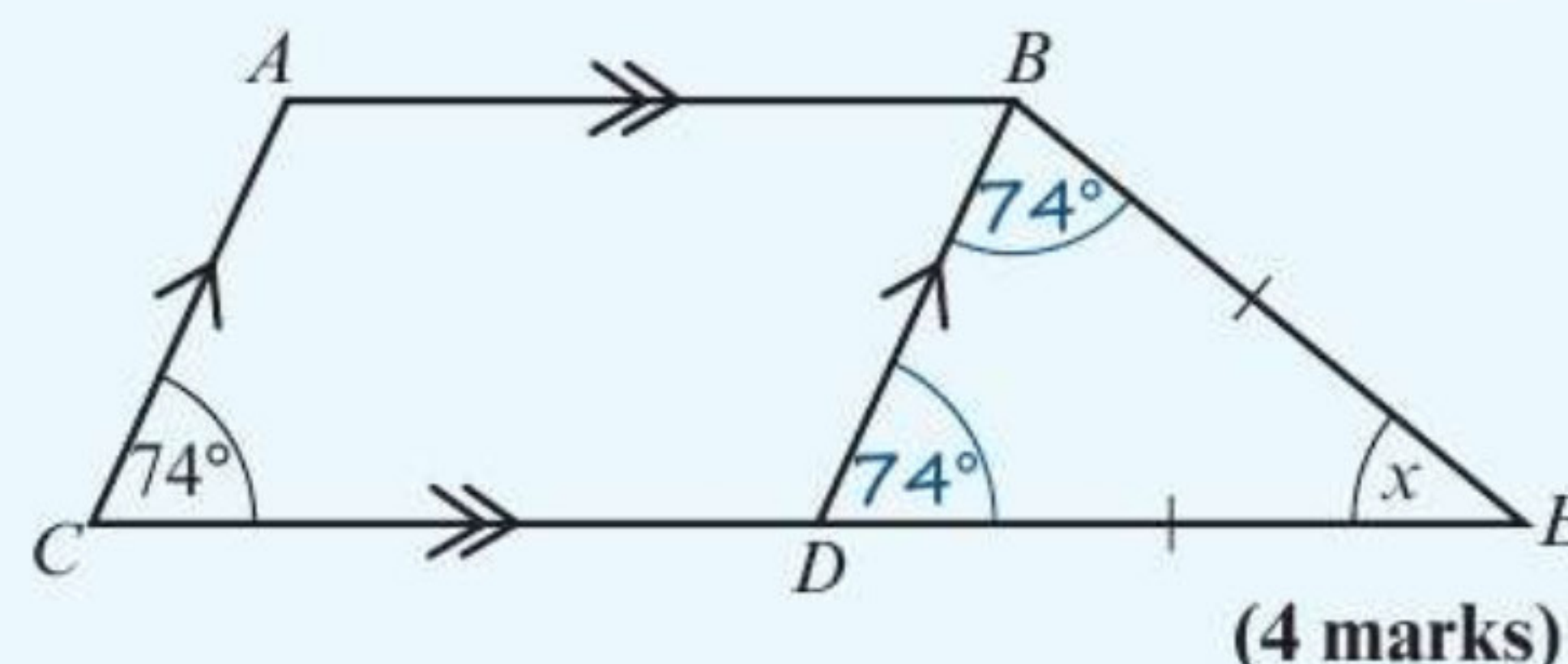
$\angle BDE = 74°$ (corresponding angles are equal)

$\angle DBE = 74°$ (base angles in an isosceles triangle are equal)
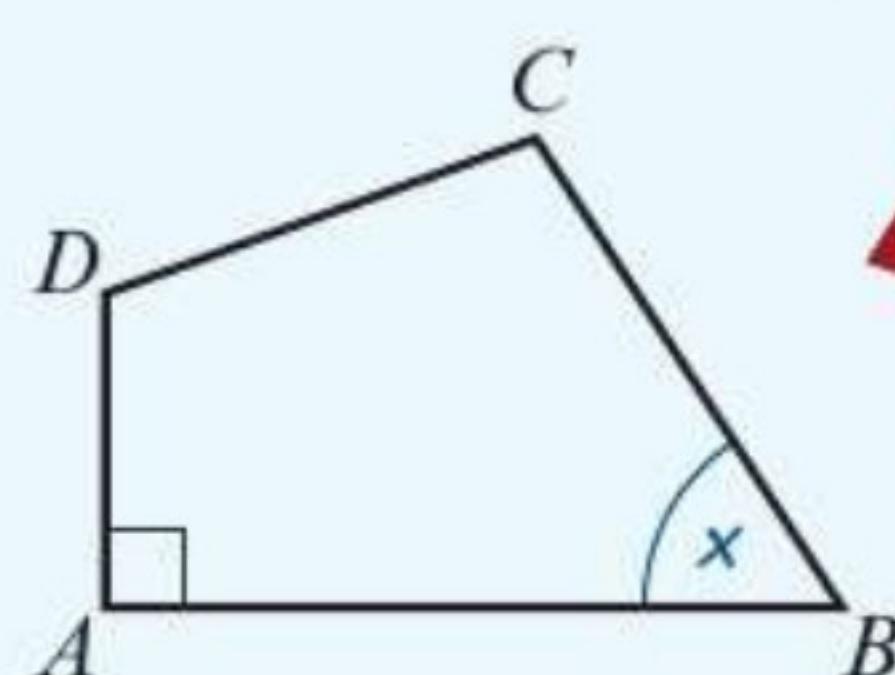
$x + 74° + 74° = 180°$ (angles in a triangle add up to 180°)

$x = 180° - 148°$

$x = 32°$

Worked example

Target grade **6**

ABCD is a quadrilateral.
Angle DAB is a right angle.
Angles ABC and BCD are in the ratio 1 : 2
Angle CDA is 70° more than angle ABC.
Work out the size of angles ABC, BCD and CDA.   **(4 marks)**

$A = 90°$

$C = 2x$

$D = x + 70°$

$A + B + C + D = 360°$ (angles in a quadrilateral add up to 360°)

So $90° + x + 2x + (x + 70°) = 360°$

$\qquad\qquad 4x = 200°$

$\qquad\qquad x = 50°$

So $B = 50°$, $C = 2 \times 50° = 100°$ and $D = 50° + 70° = 120°$

Examiners' report

Make sure you know how to use **three letter notation** for angles. The angle marked x on this diagram is angle ABC because B is the **middle letter**.

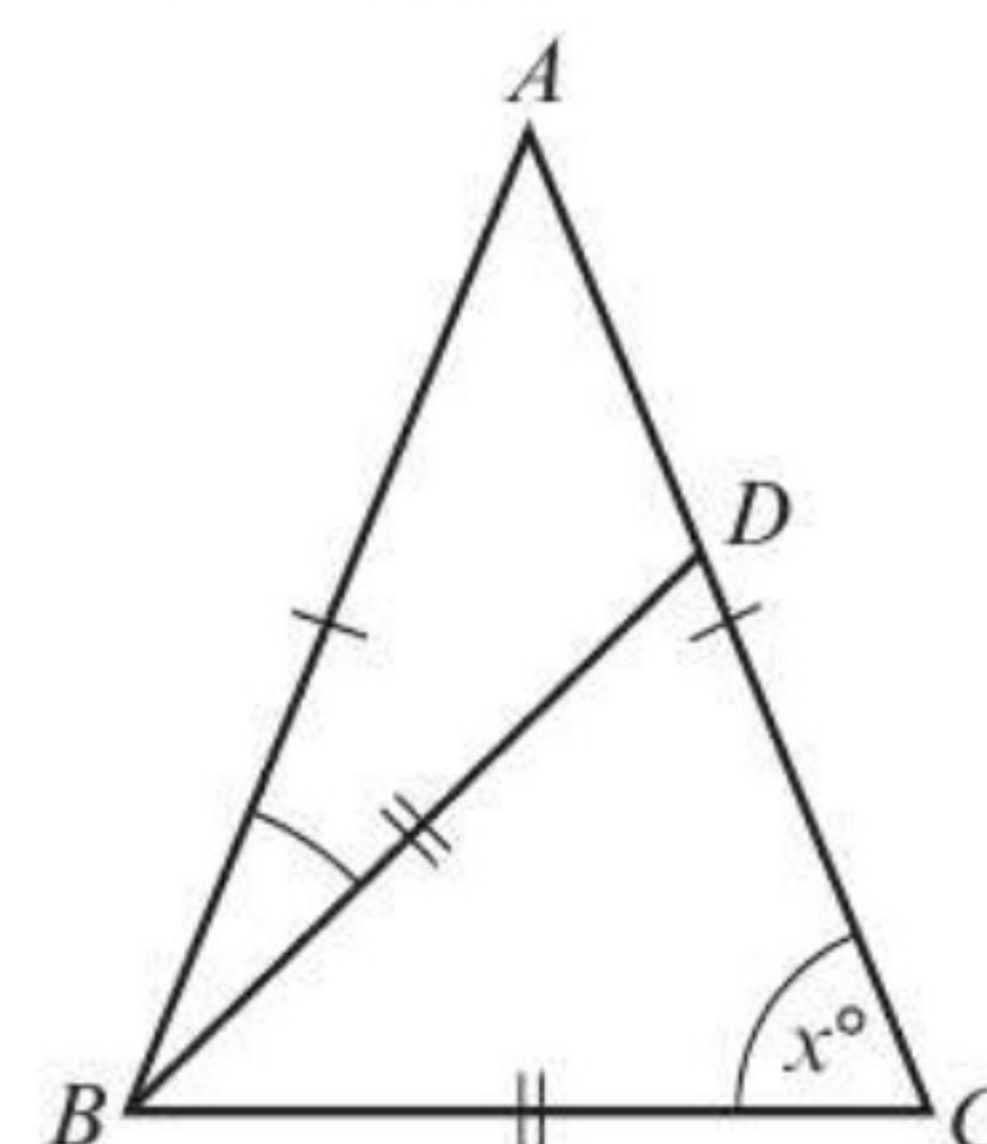Real students have struggled with questions like this in recent exams – **be prepared!**

Now try this

Target grade **6**

In the diagram ABC and BDC are isosceles triangles. Express the size of angle ABD in terms of x, giving your answer as simply as possible. Give a reason for each step of your working.   **(4 marks)**
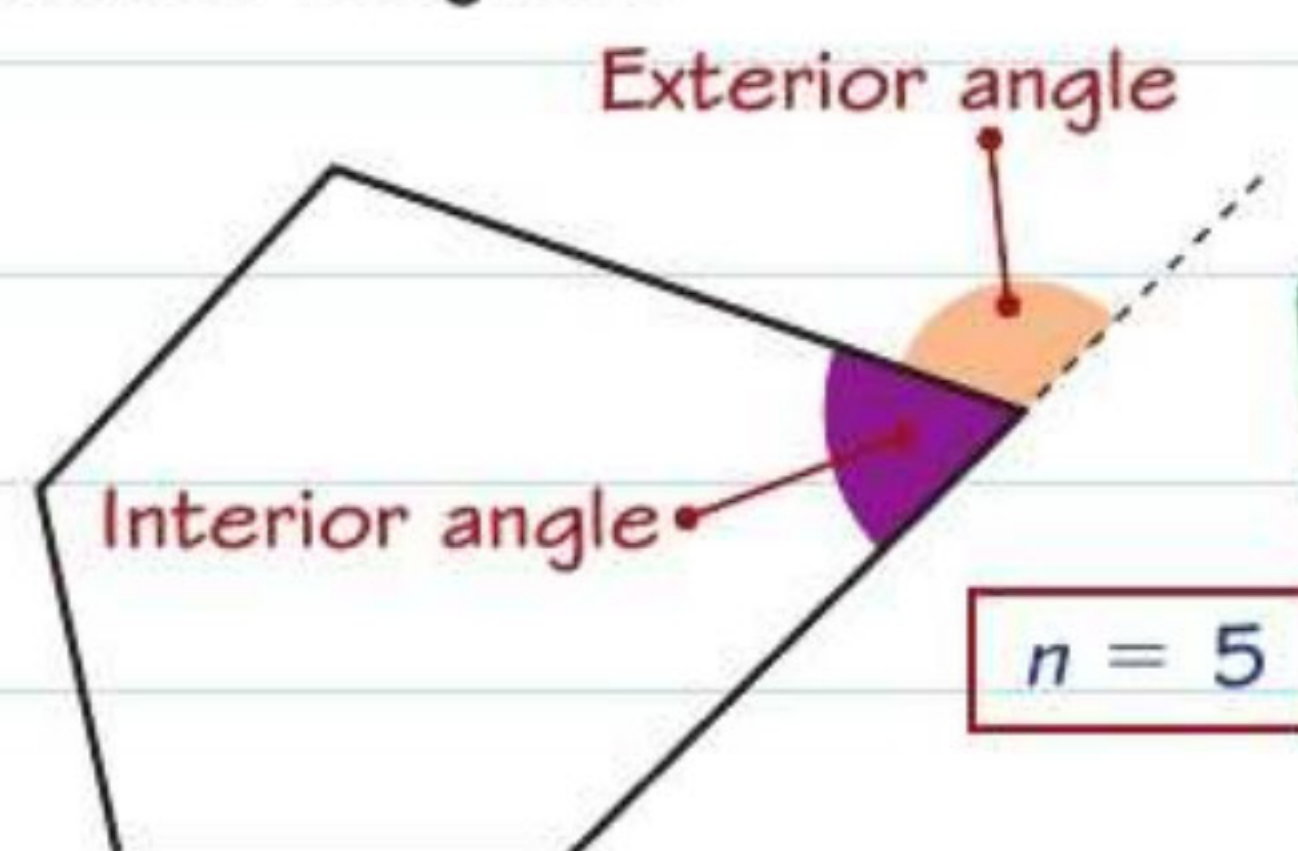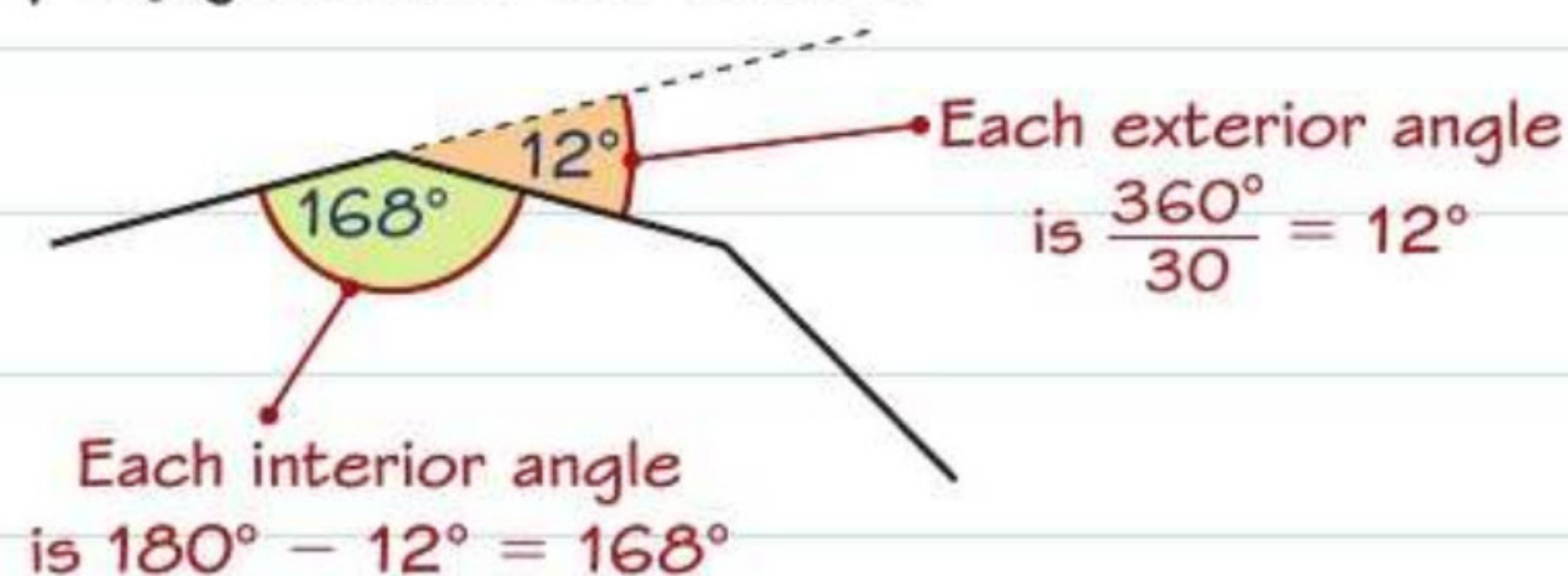
Angles in polygons

Polygon questions are all about interior and exterior angles.

Exterior angle

Interior angle

LEARN IT!

$n = 5$

Use these formulae for a polygon with n sides:
Sum of interior angles $= 180° \times (n - 2)$
Sum of exterior angles $= 360°$

This diagram shows part of a **regular** polygon with 30 sides.

168°   12°

Each exterior angle is $\frac{360°}{30} = 12°$

Each interior angle is $180° - 12° = 168°$

Don't try to draw a 30-sided polygon!
If there's no diagram given in a polygon question, you probably don't need to draw one.
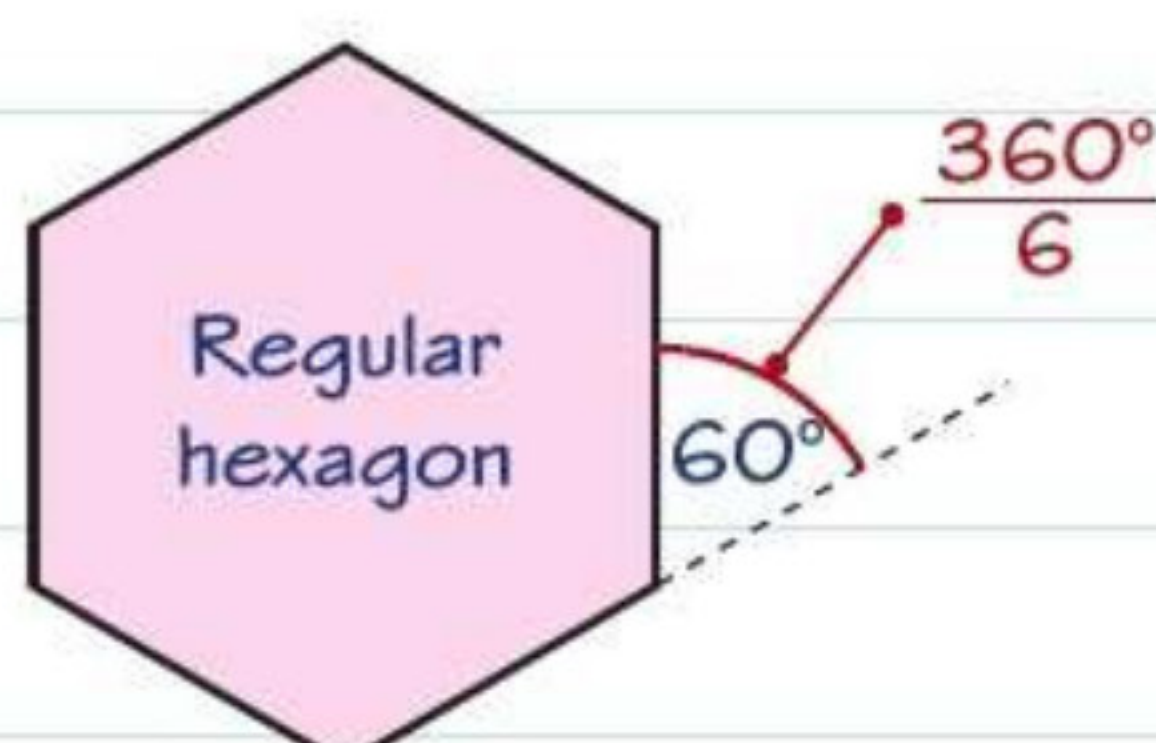

Regular polygons

In a regular polygon all the sides are equal and all the angles are equal.
If a regular polygon has n sides then each exterior angle is $\frac{360°}{n}$

LEARN IT!

Regular pentagon $\frac{360°}{5}$ 72°

Regular hexagon $\frac{360°}{6}$ 60°

Regular octagon $\frac{360°}{8}$ 45°

You can use the fact that the angles on a straight line add up to 180° to work out the size of one of the interior angles.

Worked example

Target grade 6

The diagram shows part of a regular polygon.
The interior angle and the exterior angle at a vertex are marked.
The size of the interior angle is 7 times the size of the exterior angle.
Work out the number of sides of the polygon.   **(3 marks)**

$180° \div 8 = 22.5°$
$\frac{360°}{22.5°} = 16$

The polygon has 16 sides.

Problem solved! It's usually easier to work with exterior angles in polygon questions. You can rearrange the formula for the size of an exterior angle to get:
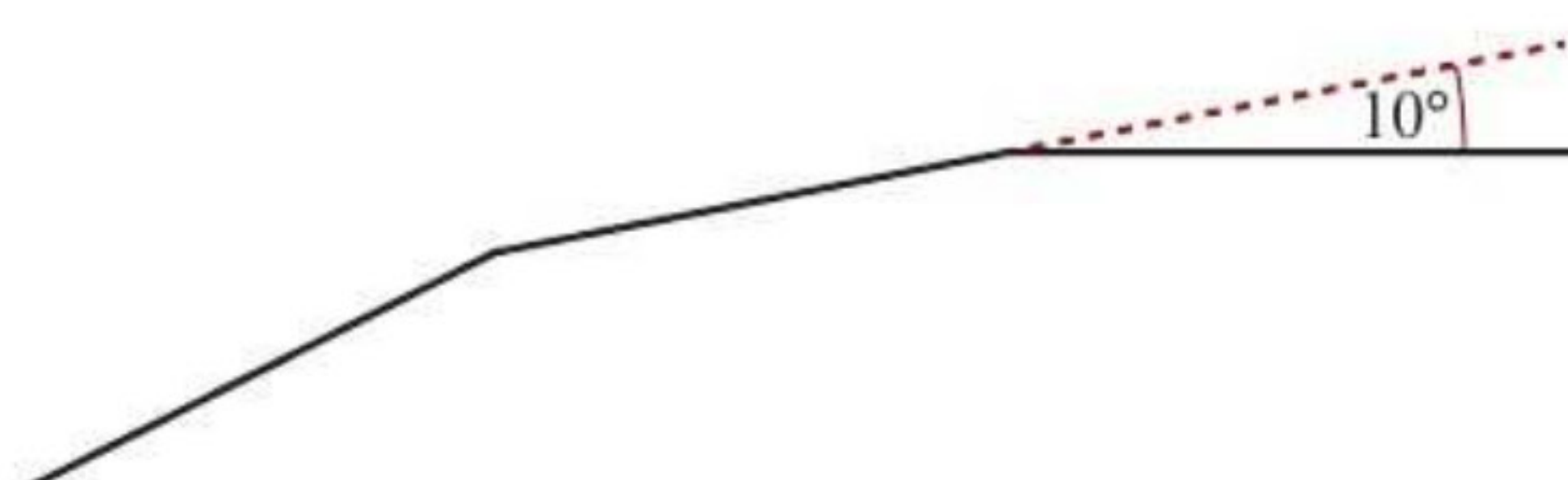$$n = \frac{360°}{\text{Exterior angle}}$$
You can use ratios to answer this question. The ratio of the interior angle to the exterior angle is 7 : 1. These angles add up to 180° so divide 180° in the ratio 7 : 1 to find the exterior angle.

Now try this

Target grade 5

The diagram shows part of a regular polygon with n sides.
(a) Work out the value of n.   **(2 marks)**
(b) What is the sum of the interior angles of the polygon?   **(2 marks)**

10°

Worked solution video

75