

Two-way tables

You can answer questions about two-way tables by adding or subtracting.

	Year 7	Year 8	Year 9	Total
Vegetarian	14	22	25	61
Not vegetarian	72	63	54	189
Total	86	85	79	250

There were 86 Year 7 students surveyed.

There were 63 non-vegetarians in Year 8.

There were 61 vegetarians in total.

In total 250 students were surveyed.

Worked example

Target grade 3

Anton surveyed 120 people about how they voted at the last general election. He recorded the results in a two-way table:

	Labour	Conservative	Other	Total
Female	21	13	13	47
Male	32	27	14	73
Total	53	40	27	120

Complete the two-way table. (4 marks)

Labour column: $53 - 21 = 32$

Female row: $47 - 21 - 13 = 13$

Conservative column: $13 + 27 = 40$

Total row: $120 - 53 - 40 = 27$

Other column: $27 - 13 = 14$

Male row: $32 + 27 + 14 = 73$

Check: $47 + 73 = 120$

$53 + 40 + 27 = 120$ ✓

Everything in blue is part of the answer.

Golden rules

The numbers in each column add up to the total for that column.

The numbers in each row add up to the total for that row.

Other
13
+ 14
= 27

Female	21	+ 13	+ 13	= 47
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To complete a two-way table:

- Write the total in the bottom right-hand cell.
- Look for rows and columns with only one missing number.
- Use addition and subtraction to find any missing values.
- Fill in the missing values as you go along.

Check it!

Add up the row totals and the column totals. They should be the same.

Now try this

Target grade 3

A photographic shop offers prints in three different sizes and on three different types of paper.

This two-way table shows information about the choices made by customers on one day.

Complete the two-way table. (4 marks)

Worked solution video



	Gloss	Matt	Lustre	Total
Small	20	35	12	
Medium	63		29	
Large		24		
Total	105		59	325

Look for rows or columns with only one missing number.

Frequency and outcomes

You might need to record all the possible **outcomes** of two or more events. You can use a **frequency tree** to show the frequencies of each possible outcome.

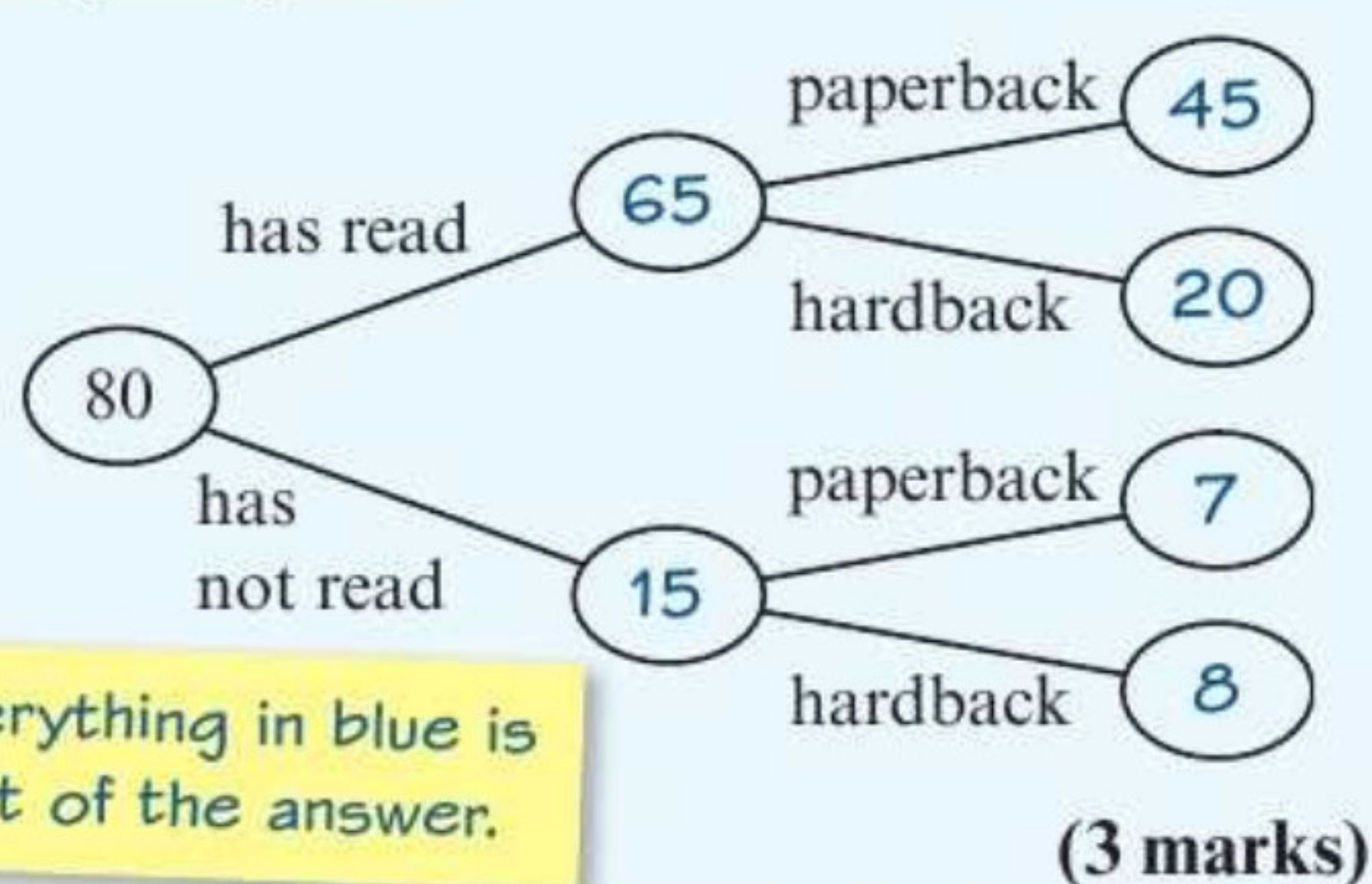
Worked example

 Target grade **3**

Wilfred owns 80 books. He has **not** read 15 of these books.

20 of the books he **has** read are hardbacks. He has 52 paperbacks in total. The rest of the books are hardbacks.

(a) Use the information to complete this frequency tree.



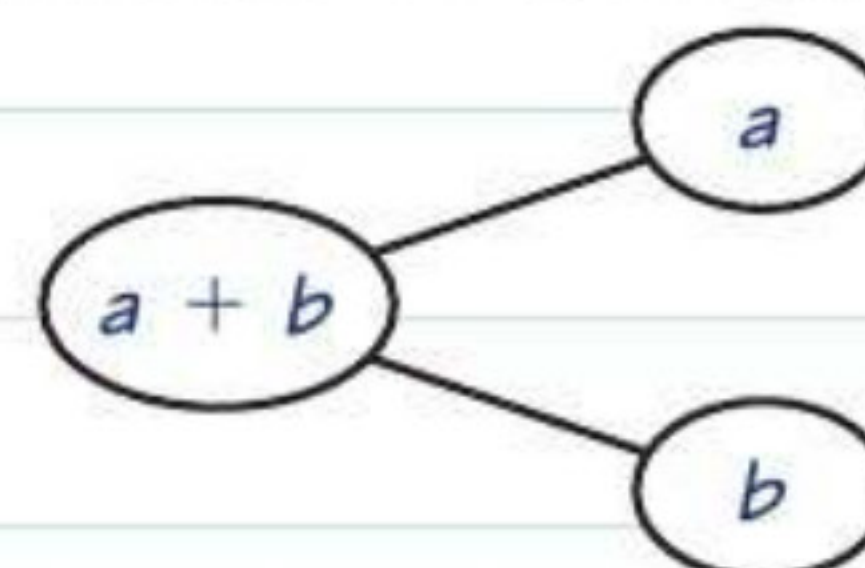
Wilfred chooses one of his books at random.

(b) Work out the probability that it is a paperback he has not read. (2 marks)

$$\frac{7}{80}$$

Golden rule

In a frequency tree, each frequency is equal to the sum of its branches.



- Write 15 in the bottom left oval, then use the golden rule above. Wilfred has read $80 - 15 = 65$ books.
- He has $80 - 52 = 28$ hardbacks in total. 20 of these are books he **has** read, so 8 must be books he has **not** read.
- Use the golden rule to complete the frequency tree.

Sample space diagrams

A **sample space diagram** shows you all the possible outcomes of two events. Here are all the possible outcomes when two coins are flipped.

		First coin	
		H	T
Second coin	H	HH	TH
	T	HT	TT

There are four possible outcomes. TH means getting a tail on the first coin and a head on the second coin.

Worked example

 Target grade **1**

This bag contains 30 counters. They are all either black or white. A counter is chosen at random. The probability that it is black is $\frac{1}{5}$. How many white counters are in the bag? (2 marks)



$$P(\text{White}) = 1 - \frac{1}{5} = \frac{4}{5}$$

$$\frac{4}{5} \times 30 = 24$$

Check it!

There are $30 - 24 = 6$ black counters in the bag.

$$P(\text{Black}) = \frac{6}{30} \checkmark$$

Now try this

Dawn is eating a meal at a restaurant. She is in a rush so she chooses one main course and one dessert from the menu at random.

- Write down all the possible outcomes. (2 marks)
- Work out the probability that Dawn chooses fish pie and cheesecake. (3 marks)

Main courses

Steak and chips
Fish pie
Mushroom pasta

Desserts

Cheesecake
Lemon mousse
Tiramisu

 Target grade **3**

Had a look ☐

Nearly there ☐

Nailed it! ☐

Probability 1

The probability that an event will happen is a value from 0 to 1.

The probability tells you how likely the event is to happen.

An event that is **certain** to happen has a probability of 1.

An event that is **impossible** has a probability of 0.

You can write a probability as a fraction, a decimal or a percentage.

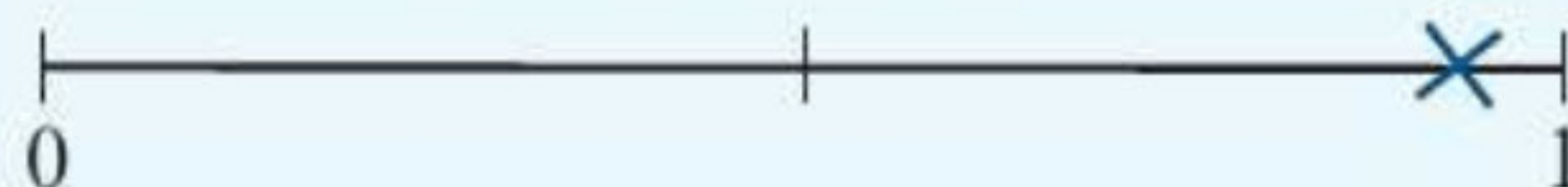


Fraction	Decimal	Percentage
$\frac{1}{2}$	0.5	50%

Worked example

Target grade 1

- (a) On this probability scale, mark with an \times the probability that it will rain in Newcastle next October. (1 mark)



- (b) Isobel says the probability she will be late for school is 7. Explain why Isobel is wrong. (1 mark)

Probabilities are numbers from 0 to 1.

It is **very likely** that it will rain in Newcastle next October. Put a cross near 1 on the probability scale.

Writing probabilities

The probability of rolling a 6 is $\frac{1}{6}$
You can write $P(6) = \frac{1}{6}$

There is one 6. There are six possible outcomes: 1, 2, 3, 4, 5, 6.



The probability of a coin landing heads up is $\frac{1}{2}$. You can write $P(\text{Heads}) = \frac{1}{2}$

There is one head. There are two possible outcomes: heads or tails.



Worked example

Target grade 1

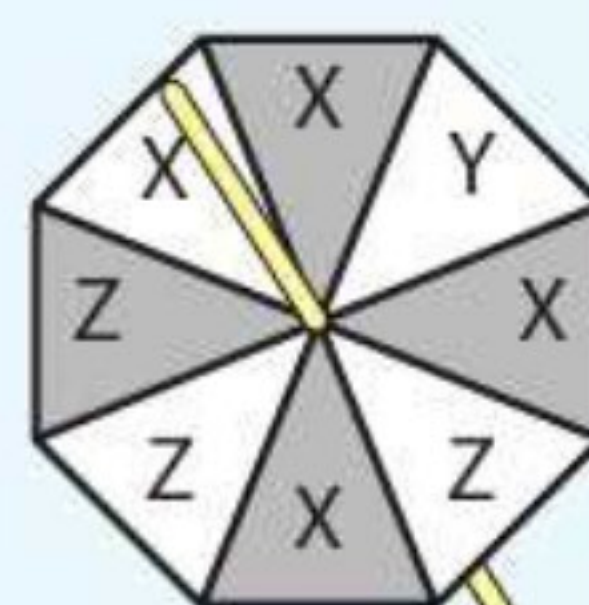
This spinner has eight equal sections.

The spinner is spun.

- (a) Write down the probability that the spinner will land on the letter Z. (1 mark)

$\frac{3}{8}$

- (b) Use a suitable probability **word** to complete this sentence:
The chance of the spinner landing on X is **evens**. (1 mark)



Golden rule

$$\text{Probability} = \frac{\text{Number of successful outcomes}}{\text{Total number of possible outcomes}}$$

Half the sections on the spinner have the letter X, so there is an even chance of landing on X. Here are some probability words you can use in your exam:

likely unlikely evens certain impossible

Now try this

Target grade 2

A letter is chosen at random from this word.



Work out the probability that the letter will be

- (a) I (1 mark) (b) S or Y or C (1 mark) (c) M. (1 mark)

Count the number of successful outcomes in each case.

Worked solution video



Probability 2

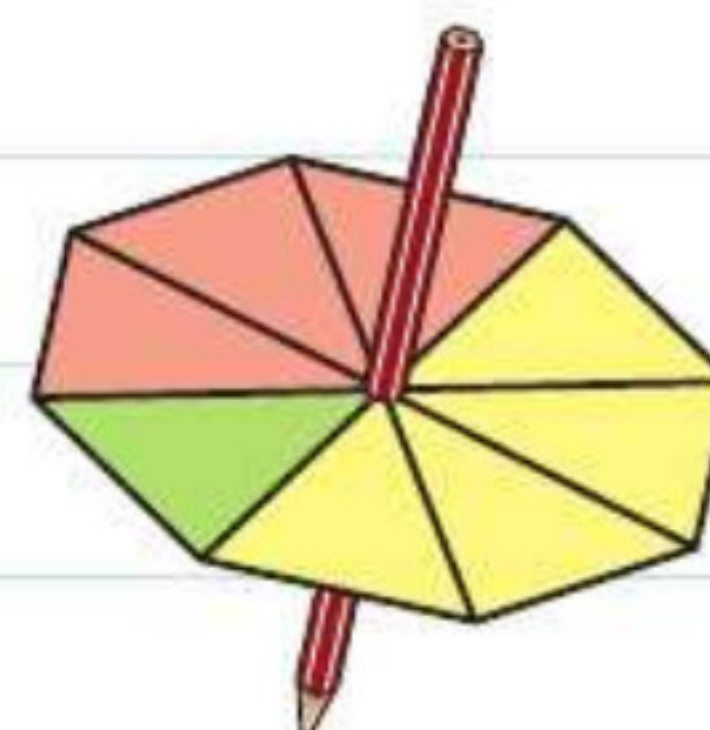
The probabilities (P) of all the different outcomes of an event add up to 1.

If you know the probability that something will happen, you can calculate the probability that it won't happen.

$$P(\text{event doesn't happen}) = 1 - P(\text{event does happen})$$



The probability of rolling a 6 on a normal fair dice is $\frac{1}{6}$. So the probability of **not** rolling a 6 is $1 - \frac{1}{6} = \frac{5}{6}$



This spinner will definitely land on either red, yellow or green. So the probability of this happening is 1.

$$P(\text{Red}) + P(\text{Yellow}) + P(\text{Green}) = 1$$

Worked example

Target grade 4

A spinner can land on red, blue, white or green. The table shows the probabilities of the spinner landing on each colour.

Colour	Red	Blue	White	Green
Probability	0.12	x	$2x$	0.28

Work out the value of x . (3 marks)

$$\begin{aligned} 0.12 + x + 2x + 0.28 &= 1 \\ 0.4 + 3x &= 1 & (-0.4) \\ 3x &= 0.6 & (\div 3) \\ x &= 0.2 \end{aligned}$$


Problem solved!

The probabilities have to add up to 1. Use this fact to write an equation, then solve it to work out the value of x .

For a reminder about solving linear equations have a look at page 30.

Check it!

$$\begin{aligned} P(\text{Blue}) = x &= 0.2 & P(\text{White}) = 2x &= 0.4 \\ 0.12 + 0.2 + 0.4 + 0.28 &= 1 \quad \checkmark \end{aligned}$$

Expectation

If you flip a coin 100 times, you can expect to get heads about 50 times. You probably won't get heads exactly 50 times, but it's a good guess.

$$\text{Expected number of outcomes} = \text{Number of trials} \times \text{Probability}$$

You can use expectation to help you decide if a dice or coin is **fair**. These two coins have been flipped 50 times each.

Coin 1

Head	
Tail	

About the same number of heads and tails. This coin is probably **fair**.

Coin 2

Head	
Tail	

A lot more than the expected number of tails. This coin is probably **biased**.

Now try this

Preti has a packet of sweets. The sweets are red, yellow, green or orange.

Preti picks one sweet at random. This table gives the probability that the sweet will be red, yellow or orange.

Colour	Red	Yellow	Green	Orange
Probability	0.13	0.36		0.28

Complete the table by working out the probability that the sweet will be green.

(2 marks)

Worked solution video


Target grade 1

Relative frequency

You need to be able to calculate probabilities for data given in graphs and tables. You can use this formula to estimate a probability from a frequency table:

$$\text{Probability} = \frac{\text{Frequency of outcome}}{\text{Total frequency}}$$

When a probability is calculated like this it is sometimes called a **relative frequency**.

Golden rule

Probability estimates based on relative frequency are **more accurate** for larger samples (or for more trials in an experiment).

Examiners' report

In the sample there were $15 + 10 = 25$ eggs which weighed 55 g or more, out of a total of 40 eggs. Don't round any values here – your answer is an **estimate** because it is based on a sample.

Real students have struggled with questions like this in recent exams – **be prepared!**



Worked example

Target grade 4

An egg farm weighed a sample of 40 eggs. It recorded the results in a frequency table.

Weight, w (g)	Frequency
$45 \leq w < 50$	6
$50 \leq w < 55$	9
$55 \leq w < 60$	15
$60 \leq w < 65$	10

- (a) Roselle buys some eggs from the farm and picks one at random. Estimate the probability that the egg weighs 55 g or more. **(2 marks)**

$$P(w \geq 55) \approx \frac{25}{40}$$

- (b) Comment on the accuracy of your estimate. **(1 mark)**

40 is a fairly small sample size, so the estimate is not very accurate.

Experimental probability

You can carry out an experiment to estimate the probability of something happening. This table shows the results of throwing a drawing pin 60 times.

Number of trials	10	20	30	40	50	60
Frequency of landing point up	8	11	17	25	30	37



To estimate the probability that the drawing pin will land point up, you calculate the relative frequency. The most accurate estimate will be based on the largest number of trials.

Now try this

Target grade 4

A four-sided dice is rolled 40 times. The results are shown in the table.

Number	1	2	3	4
Frequency	8	4	21	7



- (a) Work out the estimated probability of getting a 3. **(1 mark)**
- (b) Work out the theoretical probability of getting a 3 on a fair four-sided dice. **(1 mark)**
- (c) Do you think this dice is fair? Give a reason for your answer. **(1 mark)**

Worked solution video



If a dice is fair, the experimental probability gets closer to the theoretical probability as the number of trials increases.