Had a look ☐ Nearly there ☐ Nailed it! ☐

Fractions

You need to be able to work confidently with fractions, with or without a calculator.

1 Dividing objects

You can use fractions to divide an object into parts.

$\frac{2}{3}$ of this rectangle is shaded.

The top number is called the **numerator**.

The bottom number is called the **denominator**.

2 Equivalent fractions

Different fractions can describe the same amount.

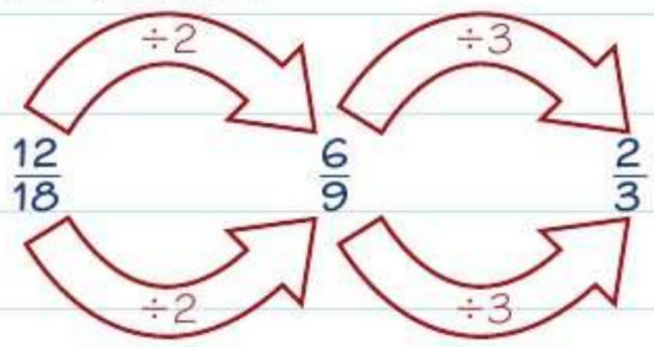
 $\frac{1}{2} = \frac{2}{4}$ 

$\frac{1}{2}$ and $\frac{2}{4}$ are called equivalent fractions.

You can find equivalent fractions by multiplying or dividing the numerator and denominator by the same number.

3 Cancelling fractions

To **cancel** or **reduce** a fraction you divide the top and bottom by the same number.



When you can't cancel the fraction any further it is in its **simplest form**.

4 Finding a fraction of an amount

Divide the amount by the denominator

↓

Multiply by the numerator

Work out $\frac{3}{10}$ of 200 kg:

200 kg ÷ 10 = 20 kg

20 kg × 3 = 60 kg

To see how to convert between fractions and decimals look at page 56.

Worked example

Target grade 1

(a) Write $\frac{20}{80}$ as a fraction in its simplest form.

$\frac{20}{80} = \frac{2}{8} = \frac{1}{4}$ **(1 mark)**

(b) Work out $\frac{2}{5}$ of £240.

240 ÷ 5 = 48

48 × 2 = 96

$\frac{2}{5}$ of £240 is £96 **(2 marks)**

Examiners' report

It is **much easier** to find equivalent fractions if you are confident with your times tables. Write down all the steps when you are simplifying.

Real students have struggled with questions like this in recent exams – **be prepared!**

$\frac{2}{5}$ is less than 1 so the answer should be less than £240. ✓

Now try this

Target grade 2

Plan your strategy before you start. Work out how much money he makes in total, compare this with £8 and then write a conclusion.

1 David buys 80 chocolates for £20.
He sells $\frac{3}{4}$ of the chocolates for 40p each.
He then sells the remaining chocolates for 20p each.
Work out the total profit that David makes. **(4 marks)**

2 Sandeep buys some pencils to sell at the school fete.
He buys 40 pencils for £8.
He sells a quarter of the pencils for 25p each and sells half of the pencils for 30p each.
The remaining pencils are broken and are thrown away.
Does Sandeep make a profit or loss?
You must show your working. **(4 marks)**

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Operations on fractions

Make sure you can add, subtract, multiply and divide fractions **without** a calculator.

1 Adding or subtracting

Write both fractions as equivalent fractions with the same denominator

↓

Add or subtract the numerators

↓

Do not change the denominator

18 is the lowest common multiple (LCM) of 9 and 6. This is the easiest common denominator to use.

For a reminder about LCMs see page 12.

Worked example Target grade 2

Work out

(a) $\frac{1}{5} + \frac{3}{10}$ **(2 marks)**

$= \frac{2}{10} + \frac{3}{10}$

$= \frac{5}{10} = \frac{1}{2}$

(b) $\frac{8}{9} - \frac{1}{6}$ **(2 marks)**

$= \frac{16}{18} - \frac{3}{18}$

$= \frac{13}{18}$

2 Multiplying

Write any whole numbers on their own as fractions with denominator 1

↓

Multiply the numerators and multiply the denominators

Worked example Target grade 2

Work out

(a) $\frac{2}{3} \times \frac{7}{10}$ **(2 marks)**

$= \frac{2 \times 7}{3 \times 10}$

$= \frac{14}{30} = \frac{7}{15}$

(b) $3 \times \frac{2}{11}$ **(2 marks)**

$= \frac{3}{1} \times \frac{2}{11}$

$= \frac{6}{11}$

3 Dividing

Write any whole numbers on their own as fractions with denominator 1

↓

Turn the second fraction 'upside down'

↓

Change ÷ to ×

↓

Multiply the numerators and multiply the denominators

Worked example Target grade 2

Work out

(a) $\frac{2}{5} \div \frac{3}{4}$ **(2 marks)**

$= \frac{2}{5} \times \frac{4}{3}$

$= \frac{8}{15}$

Change $\frac{3}{4}$ into $\frac{4}{3}$ and change ÷ to ×.

(b) $6 \div \frac{2}{3}$ **(2 marks)**

$= \frac{6}{1} \div \frac{2}{3}$

$= \frac{6}{1} \times \frac{3}{2}$

$= \frac{18}{2}$

$= 9$

Watch out!

1. You do not have to cancel your final answer unless the question asks you to 'give your answer in its simplest form'.
2. You can compare and order fractions by using equivalent fractions with the same denominator.

See page 13 for a reminder about equivalent fractions and simplest form.

Now try this Target grade 2

Worked solution video

1 Work out

(a) $\frac{5}{9} + \frac{1}{3}$ **(2 marks)**

(b) $\frac{3}{10} \div \frac{8}{15}$ **(2 marks)**

2 Work out

(a) $\frac{7}{8} - \frac{1}{2}$ **(2 marks)** (b) $\frac{8}{11} - \frac{3}{5}$ **(2 marks)**

3 Work out

(a) $\frac{5}{9} \times \frac{3}{10}$ **(2 marks)** (b) $\frac{8}{15} \div \frac{4}{7}$ **(3 marks)**

Had a look ☐ Nearly there ☐ Nailed it! ☐

Tricky Topic

Mixed numbers

Mixed numbers have a whole number part and a fraction part.

$3\frac{1}{4}$ This mixed number is the same as $3 + \frac{1}{4}$

Improper fractions have a numerator larger than their denominator.

$\frac{5}{2}$, $\frac{21}{5}$ and $\frac{4}{3}$ are all improper fractions.

Converting between mixed numbers and improper fractions

To convert a mixed number into an improper fraction you...

Multiply this... $3\frac{1}{4} = \frac{3 \times 4 + 1}{4} = \frac{13}{4}$ ...add it to this.

... by this... $3 \times 4 = 12$ $12 + 1 = 13$ Keep the same denominator.

To convert an improper fraction into a mixed number you...

Divide this... $\frac{23}{5} = 23 \div 5 = 4\frac{3}{5}$ Write the remainder as the numerator.

... by this. Keep the same denominator.

Golden rule

You need to write mixed numbers as improper fractions before you do any calculations.

Worked example

Target grade 4

Work out $4\frac{1}{2} \times 3$ **(2 marks)**

$= \frac{9}{2} \times \frac{3}{1}$

$= \frac{27}{2}$

$= 13\frac{1}{2}$

$4\frac{1}{2} = \frac{4 \times 2 + 1}{2} = \frac{9}{2}$

Worked example

Target grade 4

Work out $3\frac{2}{5} - 1\frac{1}{2}$ **(3 marks)**

$= \frac{17}{5} - \frac{3}{2}$

$= \frac{34}{10} - \frac{15}{10} = \frac{19}{10}$

$= 1\frac{9}{10}$

Worked example

Target grade 4

Kumar has $2\frac{1}{2}$ pizzas. He wants to share them equally between 3 people.
What fraction of a pizza does each person receive? **(2 marks)**

$2\frac{1}{2} \div 3 = \frac{5}{2} \div \frac{3}{1}$

$= \frac{5}{2} \times \frac{1}{3}$

$= \frac{5}{6}$

Each person receives $\frac{5}{6}$ of a pizza.

Examiners' report

Working with mixed numbers in word problems is tricky. Here are some helpful tips:

- Convert mixed numbers to improper fractions before calculating.
- If you are sharing then divide.
- Use common sense to check your answers. In this question, Kumar has less than 3 whole pizzas, so each person will get less than 1 whole pizza. You can use this fact to check your answer.

Real students have struggled with questions like this in recent exams – **be prepared!**

Now try this

Target grade 4

1 Work out
(a) $3\frac{1}{3} \times 1\frac{3}{4}$ **(3 marks)**
(b) $4\frac{1}{5} \div 1\frac{7}{8}$ **(3 marks)**

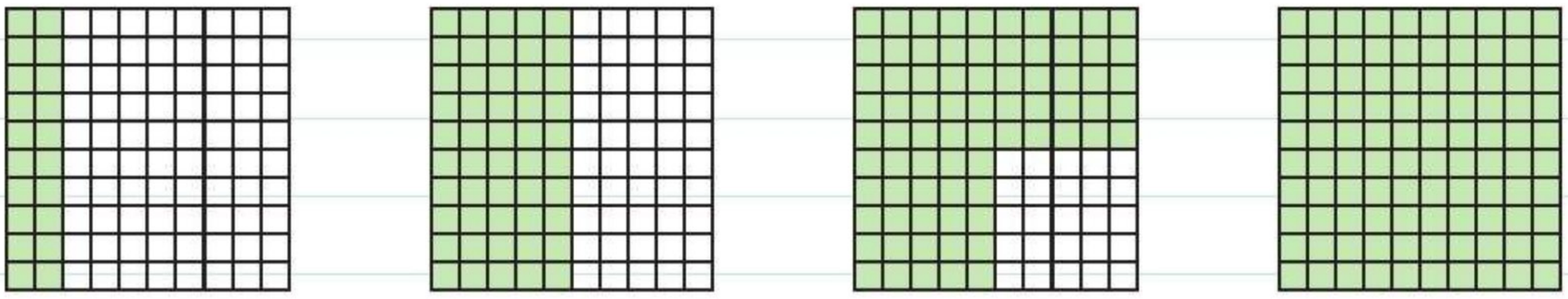
Write both numbers as improper fractions before you do the calculation. Remember you can't use a calculator for these questions.

2 A cup holds $\frac{1}{3}$ litre of water.
How many **full** cups can be filled from a $2\frac{1}{2}$ litre jug of water? **(3 marks)**

Had a look ☐ Nearly there ☐ Nailed it! ☐

Percentages

'**Per cent**' means '**out of 100**'. You can write a percentage as a fraction over 100.



$20\% = \frac{20}{100} = \frac{1}{5}$ $50\% = \frac{50}{100} = \frac{1}{2}$ $75\% = \frac{75}{100} = \frac{3}{4}$ $100\% = \frac{100}{100} = 1$

Percentages with a calculator

To find a percentage of an amount:

Divide the percentage by 100

↓

Multiply by the amount

For example, 12% of 80 cm is 9.6 cm.

$12 \div 100 = 0.12$

$0.12 \times 80 = 9.6$

Worked example Target grade 3

A car rental company reduces its prices by 15% in a sale.
A car normally costs £120 per week to rent.
Work out the weekly rental cost of a car in the sale. **(3 marks)**

$15 \div 100 = 0.15$

$0.15 \times 120 = 18$

$120 - 18 = 102$

The car costs £102 per week in the sale.

First work out 15% of £120. Then subtract this from £120.

You can also find 15% of £120 by working out 1% and then multiplying by 15.

To write one quantity as a percentage of another:

Divide the first quantity by the second quantity

↓

Multiply your answer by 100

For example, 3 out of 12 yoghurts in a pack are strawberry.

$3 \div 12 = 0.25$

$0.25 \times 100 = 25$

So 25% of the yoghurts are strawberry.

Worked example Target grade 3

In a year group of 96 students, 60 own a bicycle.
Express 60 as a percentage of 96. **(2 marks)**

$60 \div 96 = 0.625$

$0.625 \times 100 = 62.5$

62.5% of the students own a bicycle.

Now try this

1% of £18 200 is £182, so 3% is 3 lots of £182.

Target grade 1

1 Sam earns £18 200 a year.
He is given a pay rise of 3%.
How much is his pay rise? **(2 marks)**

Target grade 3

2 A family recycle 8 kg of waste in a week.
2.8 kg of this waste is paper.
What percentage of the recycled waste is paper? **(2 marks)**

Had a look ☐ Nearly there ☐ Nailed it! ☐

Hot Topic

Fractions, decimals and percentages

Here are three important facts about fractions, decimals and percentages:

1 You can convert a decimal to a percentage by multiplying by 100.

×100

0.6 = 60%

0.03 = 3%

÷100

2 You can write any percentage as a fraction with denominator 100.

$60\% = \frac{60}{100} = \frac{6}{10} = \frac{3}{5}$

Simplify your fraction as much as possible.

3 Remember these common fraction, decimal and percentage equivalents.

Fraction	$\frac{1}{100}$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$
Decimal	0.01	0.1	0.2	0.25	0.5	0.75
Percentage	1%	10%	20%	25%	50%	75%

You can arrange a list of fractions, decimals and percentages in order of size by changing them to the same type.

Worked example

Target grade 3

Carla has a bag of jelly beans.
15% of the jelly beans are strawberry.
$\frac{1}{4}$ of the jelly beans are pineapple.
$\frac{2}{5}$ of the jelly beans are apple.
The remaining jelly beans are cinnamon.
What percentage of the jelly beans are cinnamon? **(3 marks)**

$\frac{1}{4} = 25\%$

$\frac{2}{5} = 40\%$

$15 + 25 + 40 = 80$

$100 - 80 = 20$

20% of the jelly beans are cinnamon.

Problem solved!

You might need to combine fractions, decimals and percentages in a word problem like this. Read the **whole question** before doing any working. The answer needs to be a **percentage** so the quickest way of answering this question would be to convert both fractions into percentages.

Remember that all of the jelly beans is 100%, so work out the sum of the other percentages then subtract that from 100% to find the percentage of jelly beans that are cinnamon.

Now try this

Target grade 2

1 Write these percentages as fractions in their simplest form:
(a) 15% **(1 mark)**
(b) 68% **(1 mark)**

$15\% = \frac{15}{100} = \frac{\ldots\ldots}{20}$

2 Write these numbers in order, starting with the smallest:
0.42 $\frac{2}{5}$ 36% **(2 marks)**

Write all three numbers as decimals then compare.

Target grade 3

3 In a game, 5 players each put 12 counters on a table.
Sasha wins 35% of the counters.
Peter wins $\frac{2}{5}$ of the counters.
Haydon wins the remaining counters.
How many counters does Haydon win?
Show all of your working. **(3 marks)**

Worked solution video