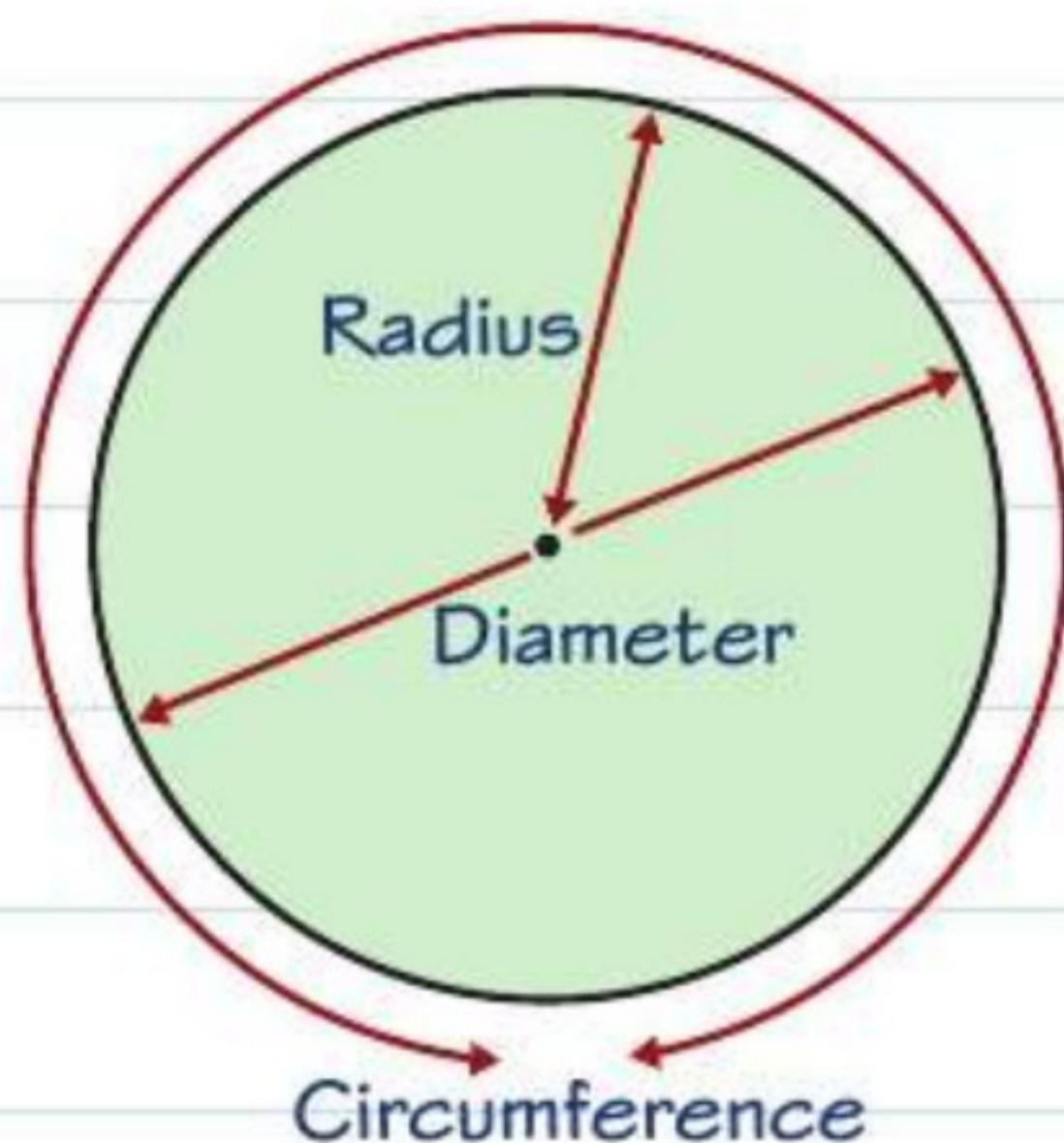


Circles



Make sure you know the definitions of **radius**, **diameter** and **circumference**.

Here are two different formulae for the circumference. You can use either.

1

$$\text{Circumference} = \pi \times \text{Diameter}$$

$$\text{Diameter} = 2 \times \text{radius}$$

2

$$\text{Circumference} = 2 \times \pi \times \text{Radius} \quad C = 2\pi r$$

LEARN IT!

π

This symbol is the Greek letter 'pi'. It always stands for the same number.

$\pi = 3.1415926\dots$

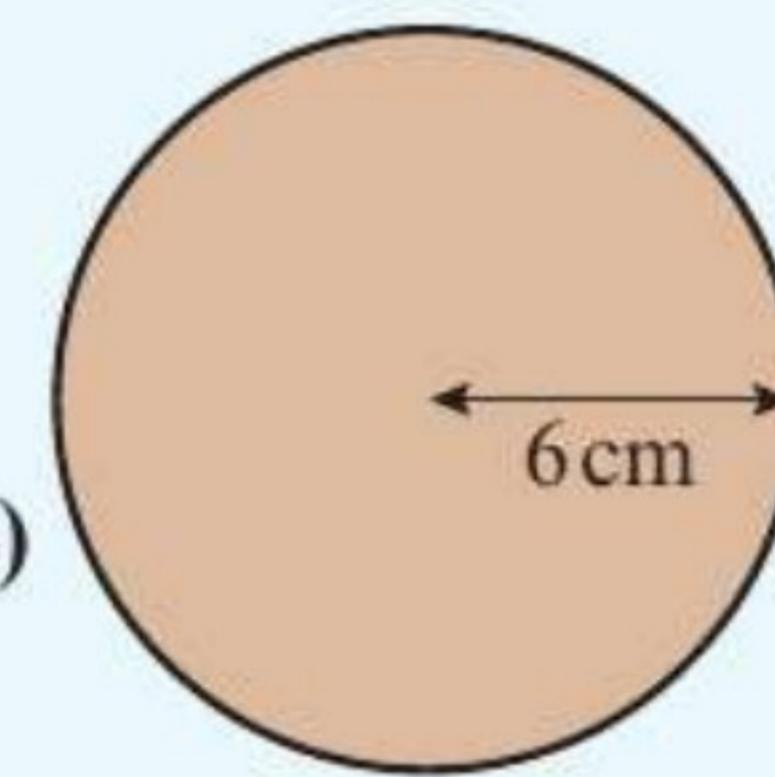
Your calculator probably has a button for entering π into a calculation. You might need to press the SHIFT key first.

If your calculator leaves π in the answer, press the $\text{S}\text{h}\text{ift}$ button to get your answer as a decimal.

You can also use 3.142 as the value of π in your exam.

Worked example

Target grade **3**



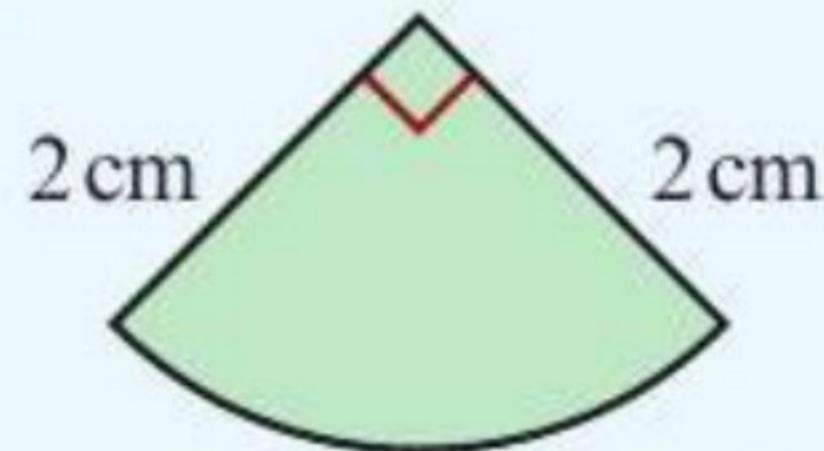
Work out the circumference of this circle.

Give your answer to 2 decimal places. **(2 marks)**

$$\begin{aligned} \text{Circumference} &= 2\pi r \\ &= 2 \times 3.142 \times 6 \\ &= 37.704 \\ &= 37.70 \text{ cm (2 d.p.)} \end{aligned}$$

Worked example

Target grade **4**



The diagram shows an earring made from a quarter of a circle.

Work out the perimeter of the earring. Give your answer to 2 decimal places.

(3 marks)

Don't round your answers until the end of your calculation.

$$\text{Circumference of whole circle} = 2\pi r$$

$$= 2 \times \pi \times 2$$

$$= 12.5663\dots$$

$$\text{Curved section of earring} = 12.5663\dots \div 4$$

$$= 3.1415\dots$$

$$\text{Total perimeter} = 2 + 2 + 3.1415\dots$$

$$= 7.1415\dots$$

$$= 7.14 \text{ cm}$$

Now try this

This steering wheel has a circumference of 120 cm.

(a) Work out the diameter of the steering wheel.

Give your answer to 1 decimal place. **(2 marks)**

(b) Work out the radius of the steering wheel.

Give your answer to 1 decimal place. **(1 mark)**

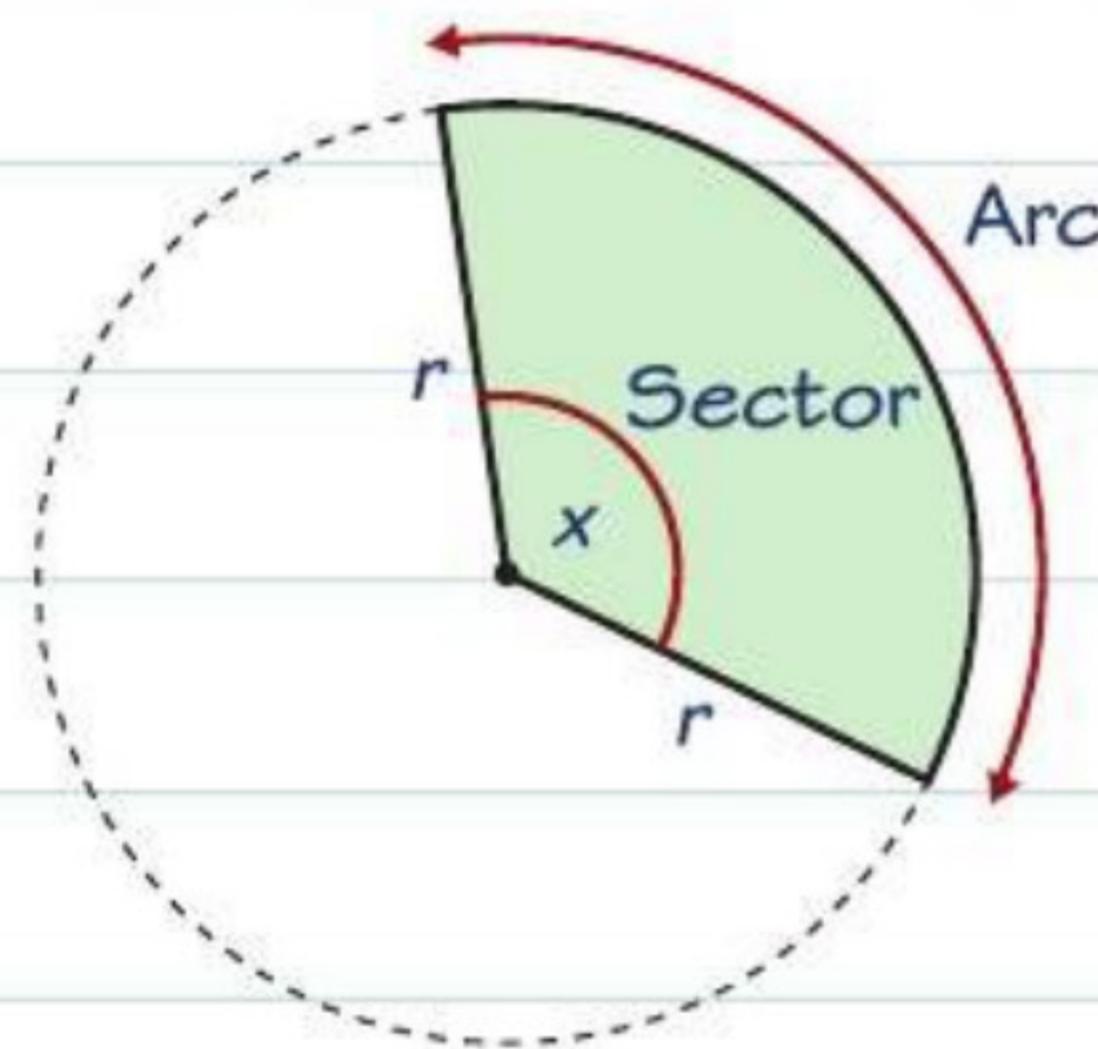


Target grade **3**

Sectors of circles

Each pair of radii divides a circle into two sectors, a **major sector** and a **minor sector**.

You can find the area of a sector by working out what fraction it is of the whole circle.



For a sector with angle x of a circle with radius r :

Sector = $\frac{x}{360^\circ}$ of the whole circle so

Area of sector = $\frac{x}{360^\circ} \times \pi r^2$

Arc length = $\frac{x}{360^\circ} \times 2\pi r$

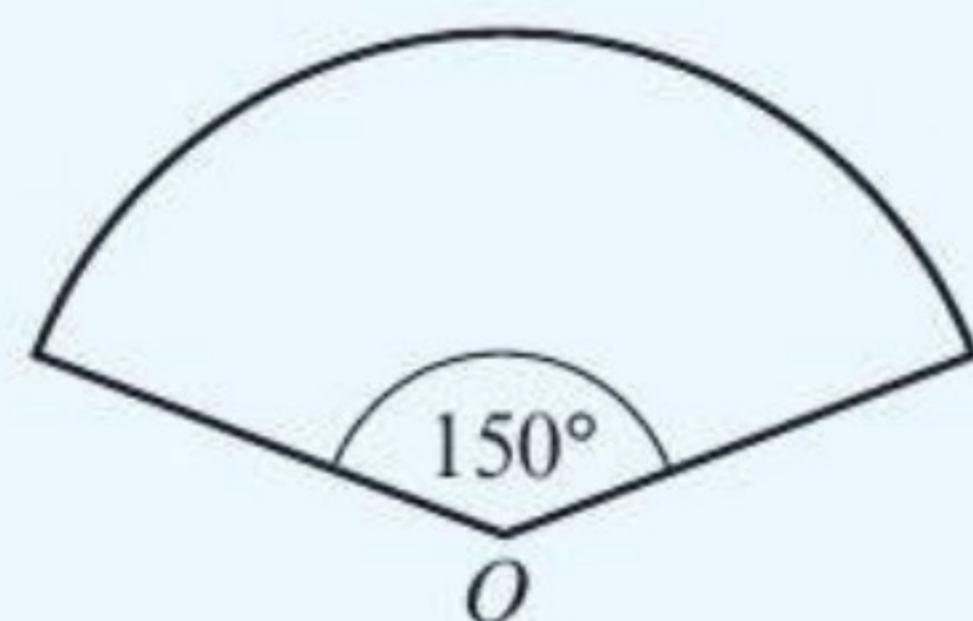
LEARN IT!

You can give answers in terms of π .
There is more about this on the next page.

Worked example

Target grade **5**

The diagram shows a minor sector of a circle of radius 13 cm.



Work out the perimeter of the sector. (4 marks)

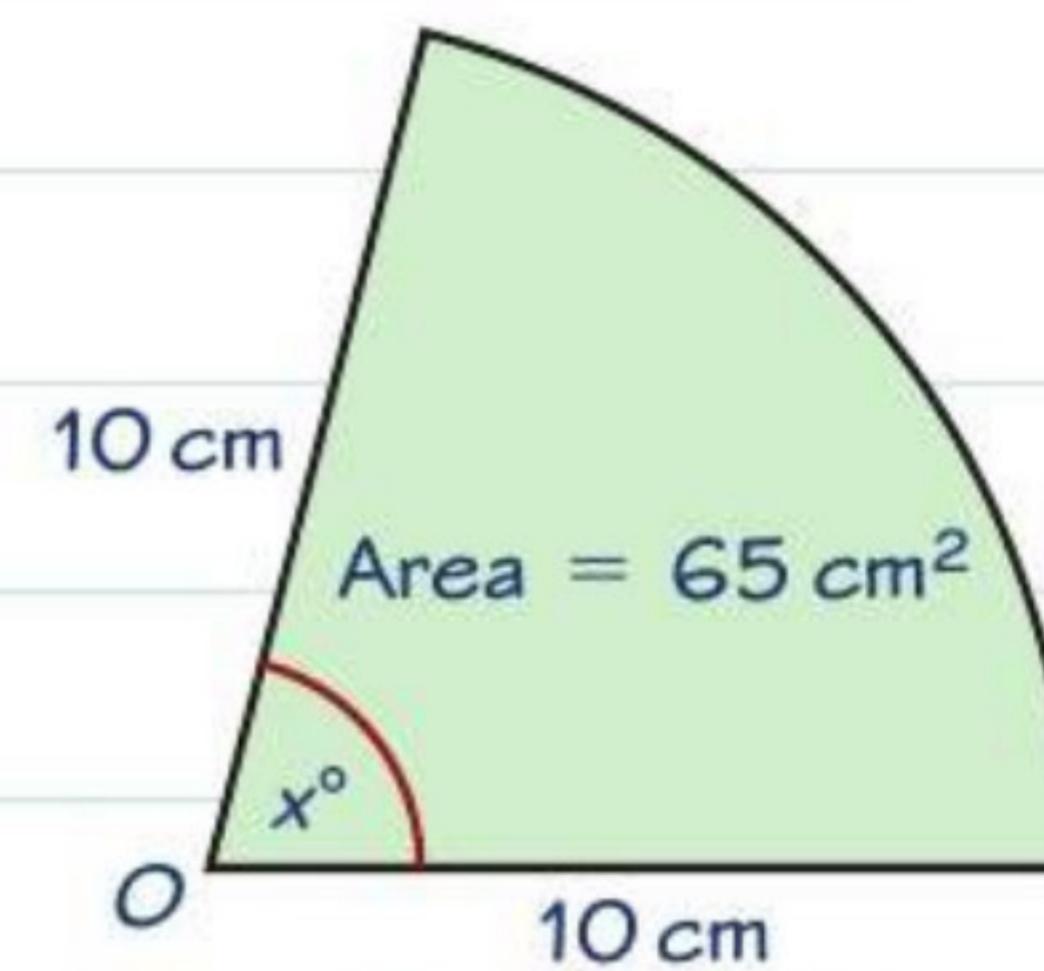
$$\begin{aligned}\text{Arc length} &= \frac{x}{360^\circ} \times 2\pi r \\ &= \frac{150^\circ}{360^\circ} \times 2 \times \pi \times 13 \\ &= 34.03392\dots\end{aligned}$$

$$\begin{aligned}\text{Perimeter} &= \text{Arc length} + \text{Radius} + \text{Radius} \\ &= 34.03392\dots + 13 + 13 \\ &= 60 \text{ cm (2 s.f.)}\end{aligned}$$

Don't round until your final answer. The radius is given correct to 2 significant figures so this is a good degree of accuracy.

Finding a missing angle

You can use the formulae for arc length or area to find a missing angle in a sector. Practise this method to help you tackle the hardest questions.



$$\begin{aligned}\text{Area of sector} &= \frac{x}{360^\circ} \times \pi r^2 \\ 65 &= \frac{x}{360^\circ} \times 100\pi \\ x &= \frac{65 \times 360}{100\pi} \\ &= 74.4845\dots \\ &= 74.5^\circ \text{ (to 3 s.f.)}\end{aligned}$$

Now try this

Target grade **5**

OAB is a sector of a circle, centre O .

Angle $AOB = 60^\circ$.

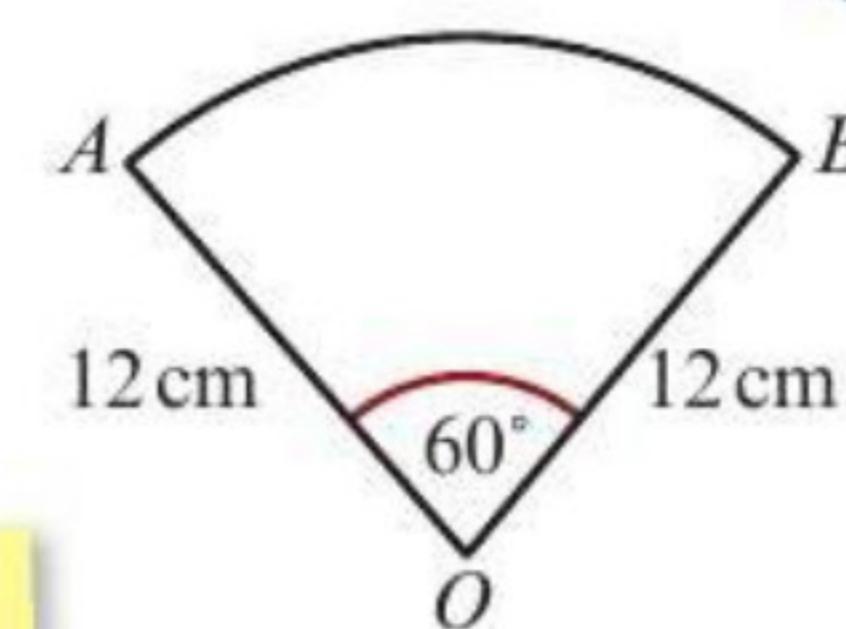
$OA = OB = 12$ cm.

Work out the length of the arc AB .

Give your answer correct to 3 significant figures.

(3 marks)

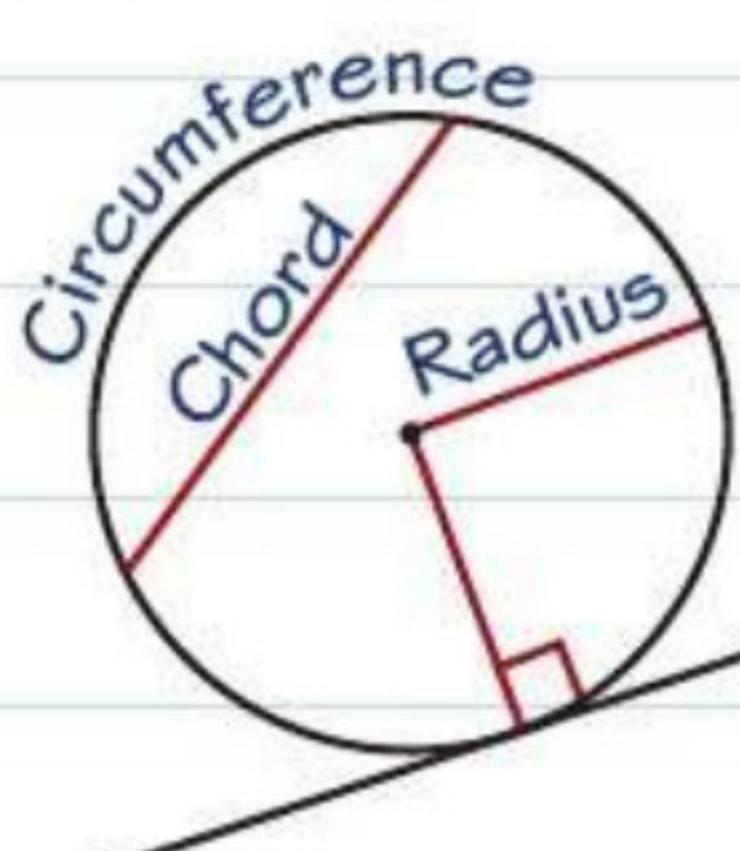
You need to learn the formula for arc length.





Circle facts

You need to know the names of the different parts of a circle.



$$\text{Diameter} = \text{Radius} \times 2$$

Tangent

The other parts of a circle are shown on pages 83 and 84.

When you are solving circle problems:

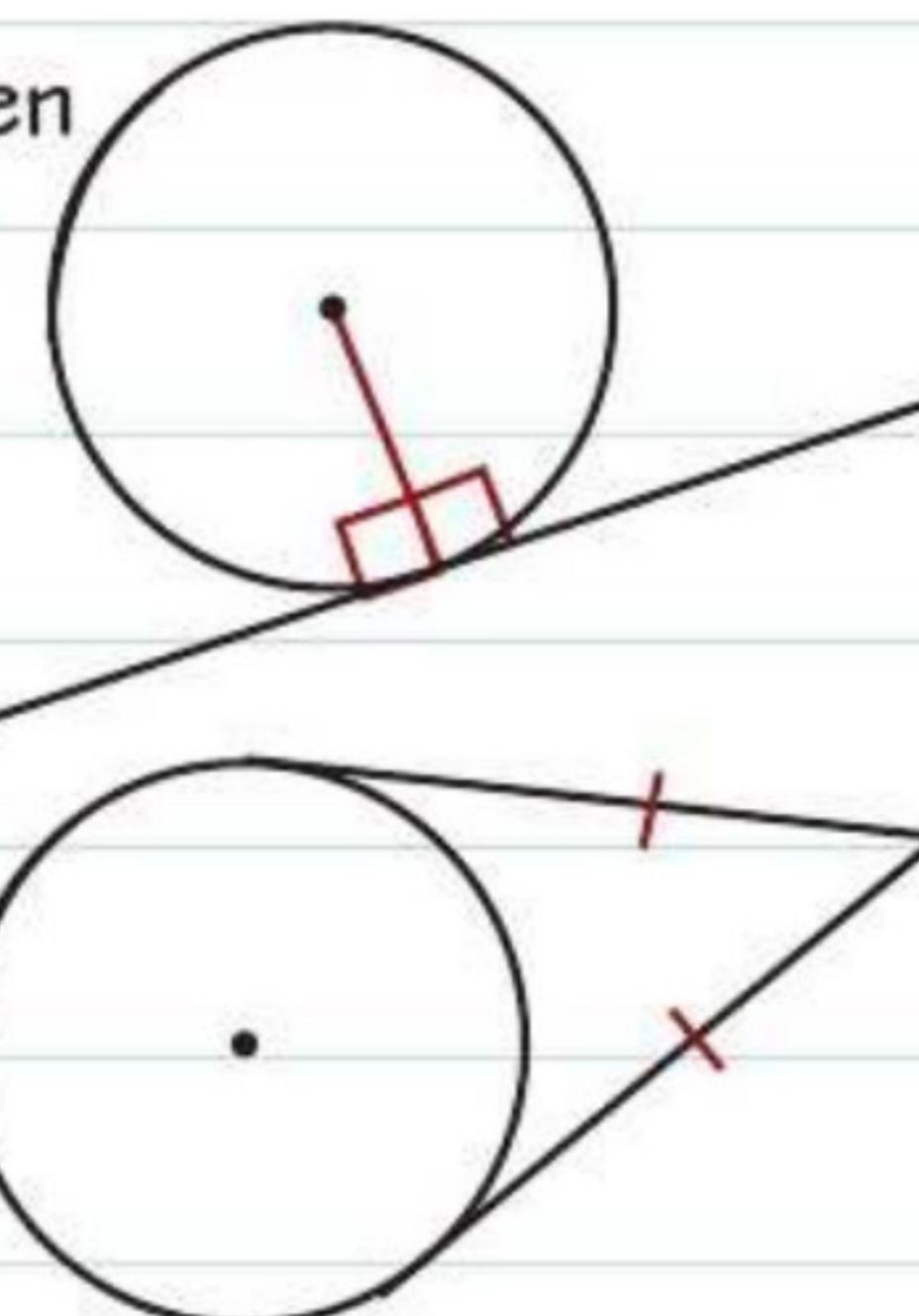
- Correctly identify the angle to be found.
- Use all the information given in the question.
- Mark all calculated angles on the diagram.
- Give a reason for each step of your working.

You might need to use angle facts about triangles, quadrilaterals and parallel lines in circle questions. There is a list of angle facts on page 74.

Key circle facts

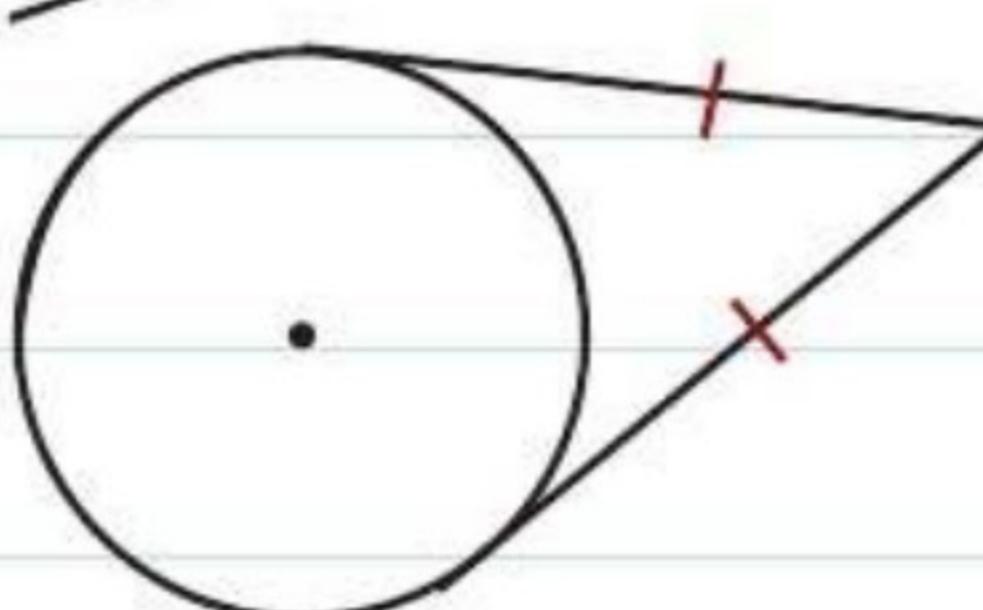
1

The angle between a radius and a tangent is 90° .



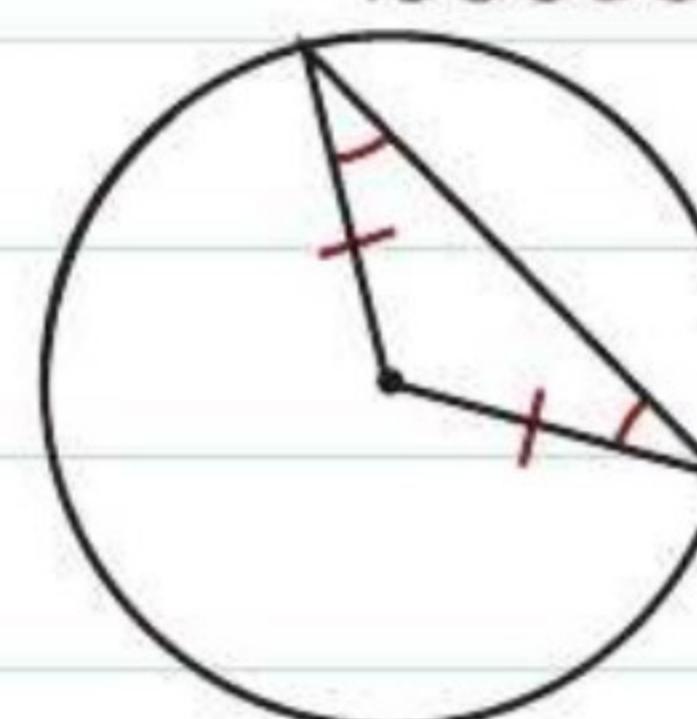
2

Two tangents which meet at a point outside a circle are the same length.



3

A triangle which has one vertex at the centre of a circle and two vertices on the circumference is an **isosceles triangle**.



Each short side of the triangle is a radius, so they are the same length.

Remember that the base angles of an isosceles triangle are equal.

Worked example

Target grade **6**

A and B are points on the circumference of a circle centre O . AC and BC are both tangents to the circle. Angle $BCA = 42^\circ$. Work out the size of the angle marked x . (3 marks)

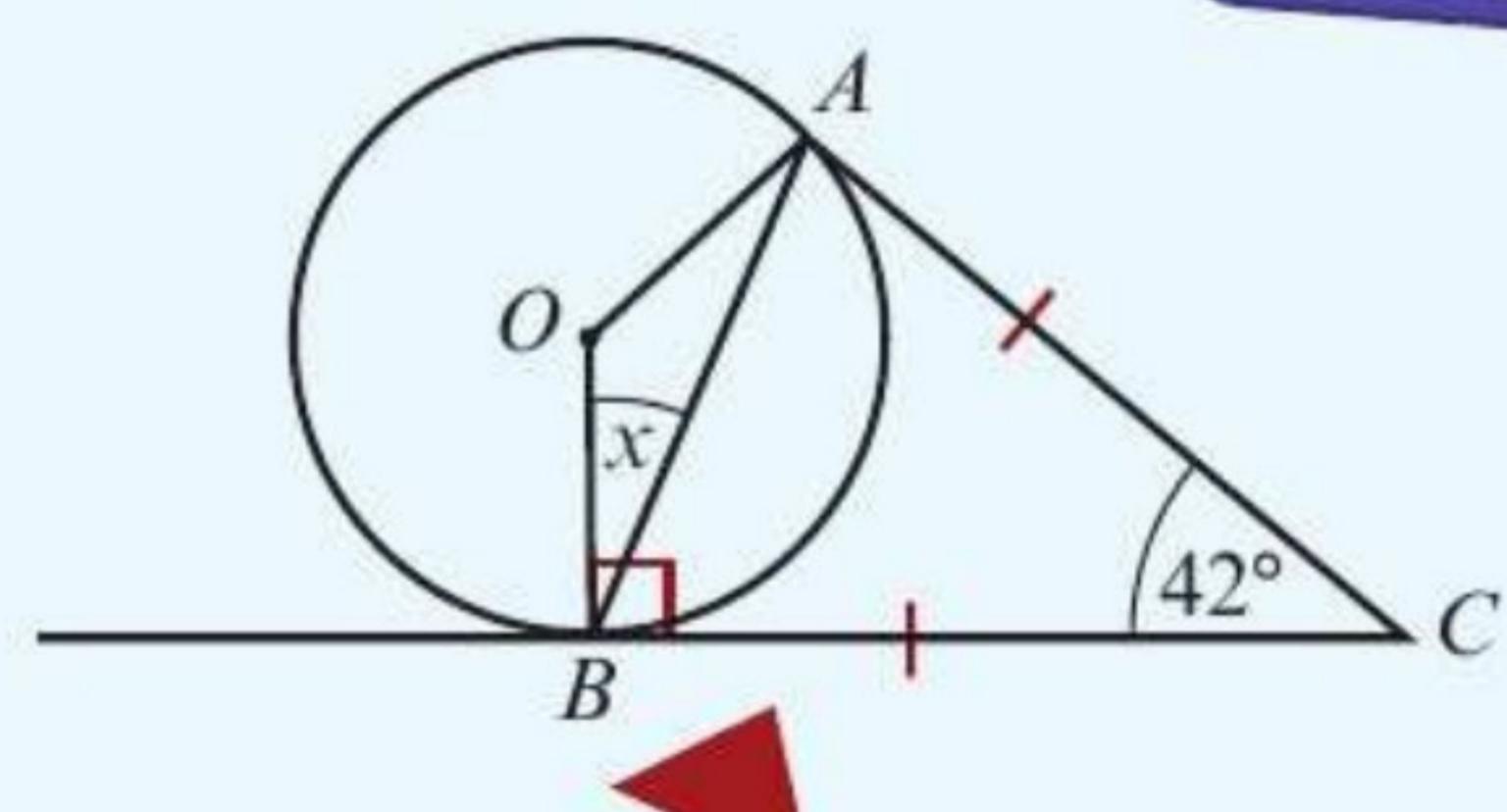
$AC = BC$ (tangents from a point outside a circle are the same length)

$$\angle ABC = \frac{180^\circ - 42^\circ}{2} = 69^\circ$$

(base angles in an isosceles \triangle are equal, and angles in a \triangle add up to 180°)

$$x + 69^\circ = 90^\circ \text{ (angle between a tangent and a radius} = 90^\circ)$$

$$x = 21^\circ$$



$AC = BC$, so mark these lines with a dash. Make sure you also write down the circle fact you are using. To write a really good answer you have to give a reason for each step of your working.

Now try this

Target grade **7**

A , B and C are points on the circumference of a circle with centre O . CD is a tangent to the circle. Angle $AOB = 53^\circ$

Work out the size of angle BCD . (3 marks)

Make sure you write down every step of your working.

