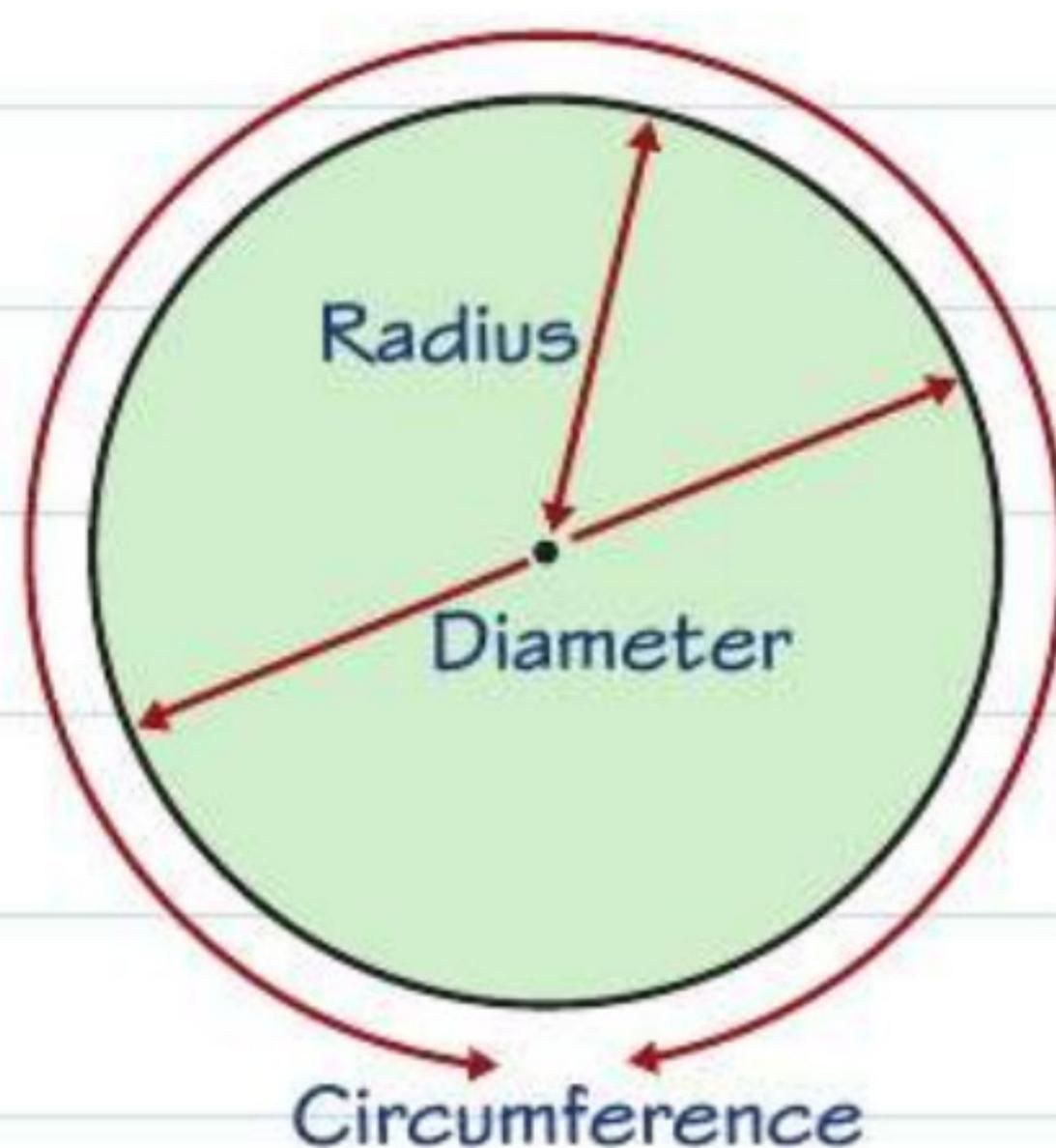


Circles



Make sure you know the definitions of **radius**, **diameter** and **circumference**.

Here are two different formulae for the circumference. You can use either.

1

$$\text{Circumference} = \pi \times \text{Diameter}$$

$$\text{Diameter} = 2 \times \text{radius}$$

2

$$\text{Circumference} = 2 \times \pi \times \text{Radius} \quad C = 2\pi r$$

LEARN IT!

π

This symbol is the Greek letter 'pi'. It always stands for the same number.

$\pi = 3.1415926\dots$

Your calculator probably has a button for entering π into a calculation. You might need to press the SHIFT key first.

If your calculator leaves π in the answer, press the $\text{S}\text{h}\text{ift}$ button to get your answer as a decimal.

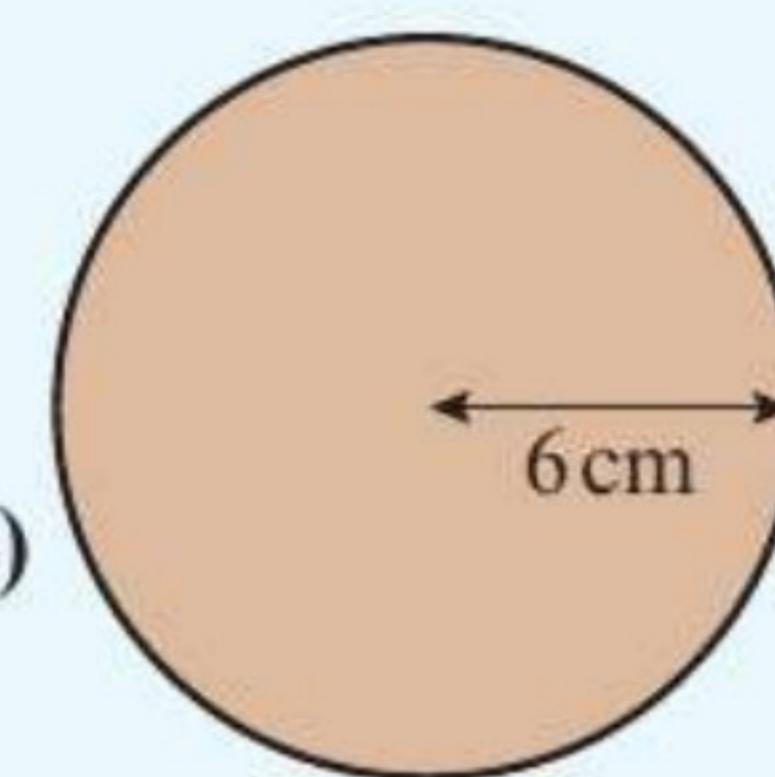
You can also use 3.142 as the value of π in your exam.

Worked example

Target grade **3**

Work out the circumference of this circle.

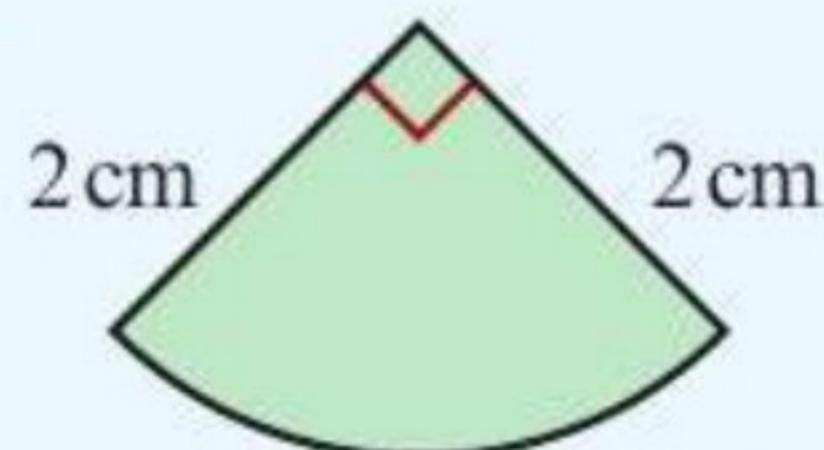
Give your answer to 2 decimal places. **(2 marks)**



$$\begin{aligned} \text{Circumference} &= 2\pi r \\ &= 2 \times 3.142 \times 6 \\ &= 37.704 \\ &= 37.70 \text{ cm (2 d.p.)} \end{aligned}$$

Worked example

Target grade **4**



The diagram shows an earring made from a quarter of a circle.

Work out the perimeter of the earring. Give your answer to 2 decimal places.

(3 marks)

Don't round your answers until the end of your calculation.

$$\begin{aligned} \text{Circumference of whole circle} &= 2\pi r \\ &= 2 \times \pi \times 2 \\ &= 12.5663\dots \end{aligned}$$

$$\begin{aligned} \text{Curved section of earring} &= 12.5663\dots \div 4 \\ &= 3.1415\dots \end{aligned}$$

$$\begin{aligned} \text{Total perimeter} &= 2 + 2 + 3.1415\dots \\ &= 7.1415\dots \\ &= 7.14 \text{ cm} \end{aligned}$$

Now try this

This steering wheel has a circumference of 120 cm.

- Work out the diameter of the steering wheel. Give your answer to 1 decimal place. **(2 marks)**
- Work out the radius of the steering wheel. Give your answer to 1 decimal place. **(1 mark)**



Worked solution video



Target grade **3**



You need to know the formula for the area of a circle:

$$\text{Area} = \pi \times \text{Radius}^2$$

$$A = \pi \times r \times r = \pi r^2$$

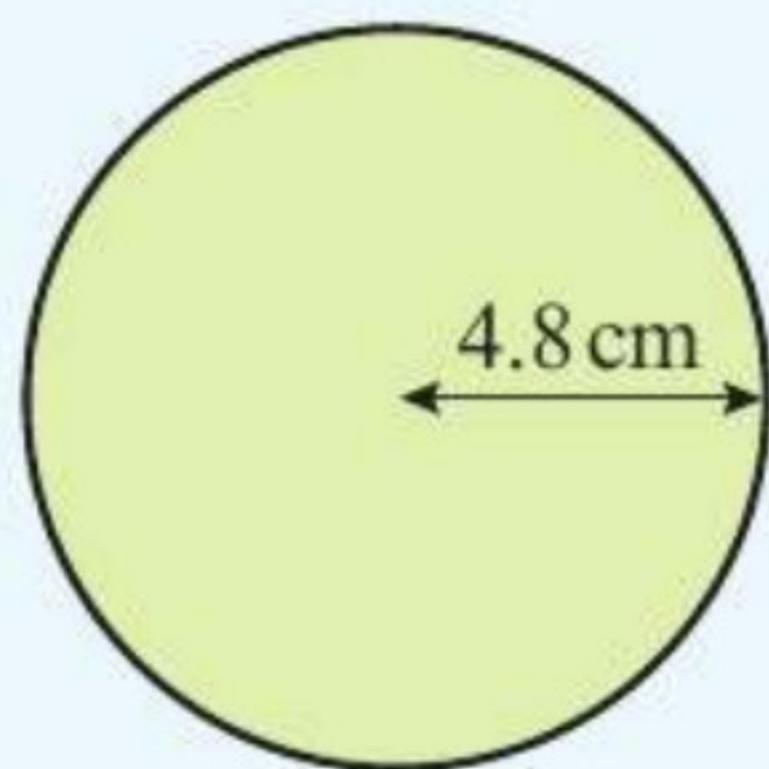
You always need to use the **radius** when you are calculating the area.

If you are given the diameter, divide it by 2 to get the radius.

LEARN IT!

Worked example

Target grade **3**



Work out the area of this circle. **(2 marks)**

$$\begin{aligned} \text{Area} &= \pi r^2 \\ &= \pi \times 4.8^2 \\ &= \pi \times 23.04 \\ &= 72.3822\dots \\ &= 72.4 \text{ cm}^2 \text{ (1 d.p.)} \end{aligned}$$

Examiners' report

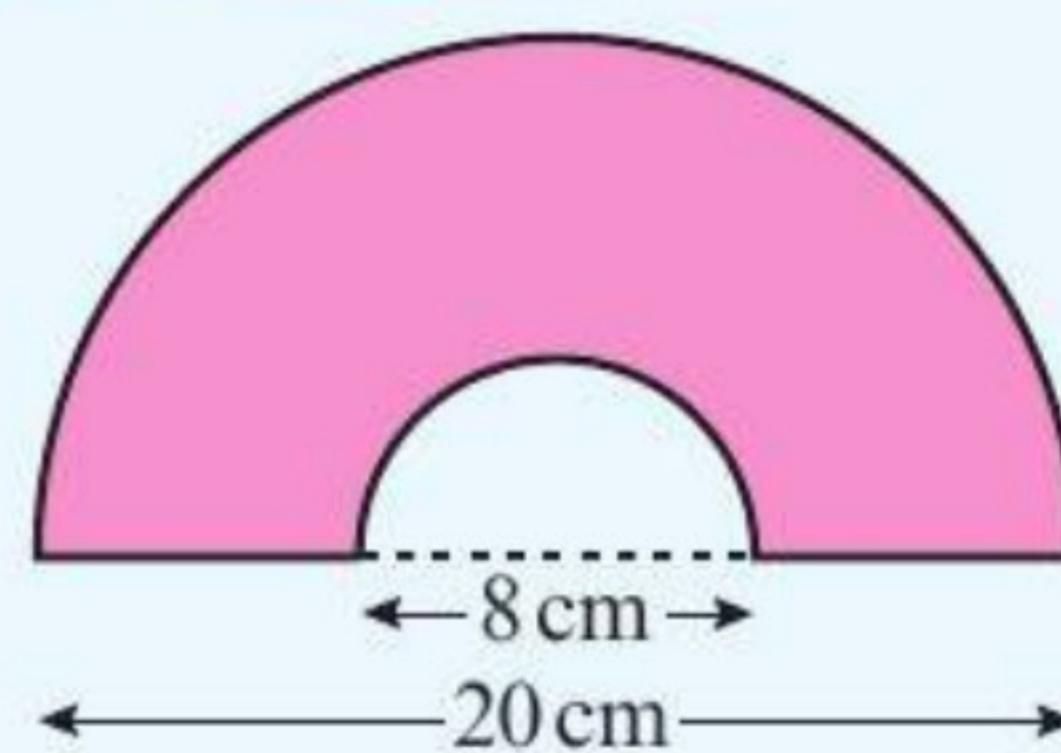
When you have to keep track of a lot of working, **write down** what each calculation represents. It will be really clear to you **and** the examiner what you have worked out!

Real students have struggled with questions like this in recent exams – **be prepared!**



Worked example

Target grade **4**



The diagram shows two semicircles with diameters 8 cm and 20 cm.

Work out the shaded area. **(4 marks)**

$$\begin{aligned} \text{Radius of large semicircle} &= 20 \div 2 \\ &= 10 \end{aligned}$$

$$\begin{aligned} \text{Area of large semicircle} &= \frac{1}{2} \times \pi \times 10^2 \\ &= 157.0796\dots \end{aligned}$$

$$\text{Radius of small semicircle} = 8 \div 2 = 4$$

$$\begin{aligned} \text{Area of small semicircle} &= \frac{1}{2} \times \pi \times 4^2 \\ &= 25.1327\dots \end{aligned}$$

$$\begin{aligned} \text{Shaded area} &= 157.0796\dots - 25.1327\dots \\ &= 131.946\dots \\ &= 132 \text{ cm}^2 \text{ (3 s.f.)} \end{aligned}$$

Examiners' report

The formula for the area of a circle uses the **radius**. If you are given the **diameter** you will need to divide by 2 first.

Real students have struggled with questions like this in recent exams – **be prepared!**



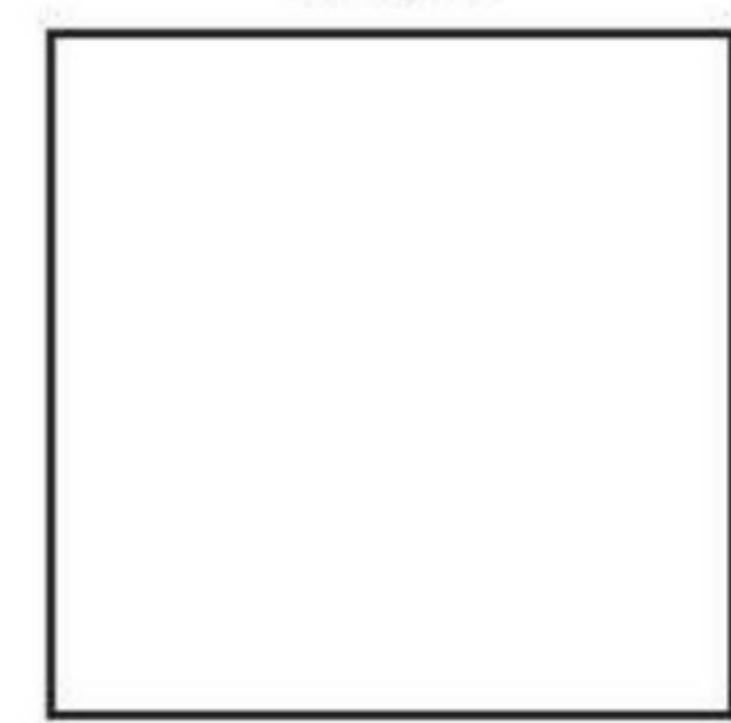
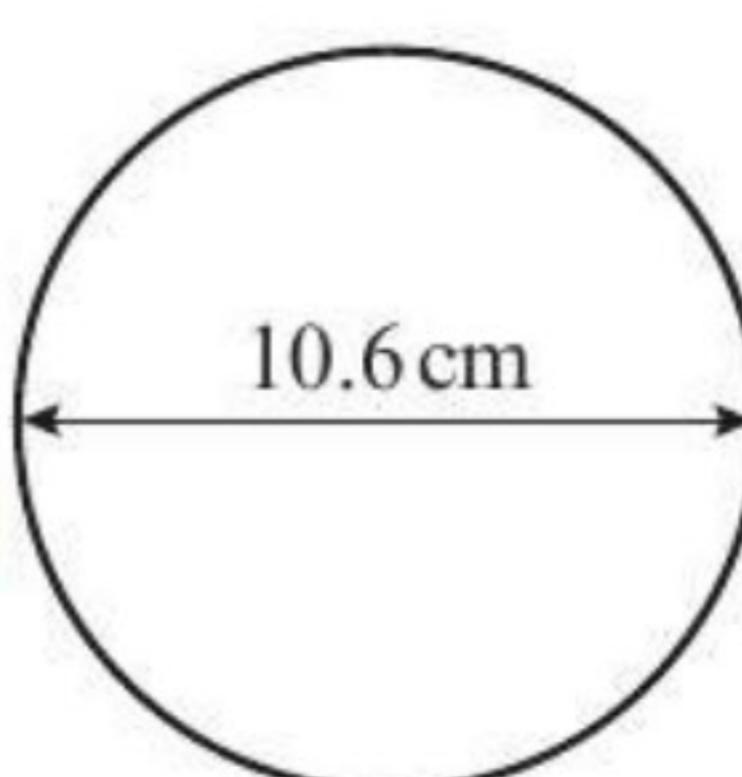
Now try this

Target grade **4**

This circle and this square are equal in area. Work out the length of the side of the square. Give your answer correct to 1 decimal place.

(4 marks)

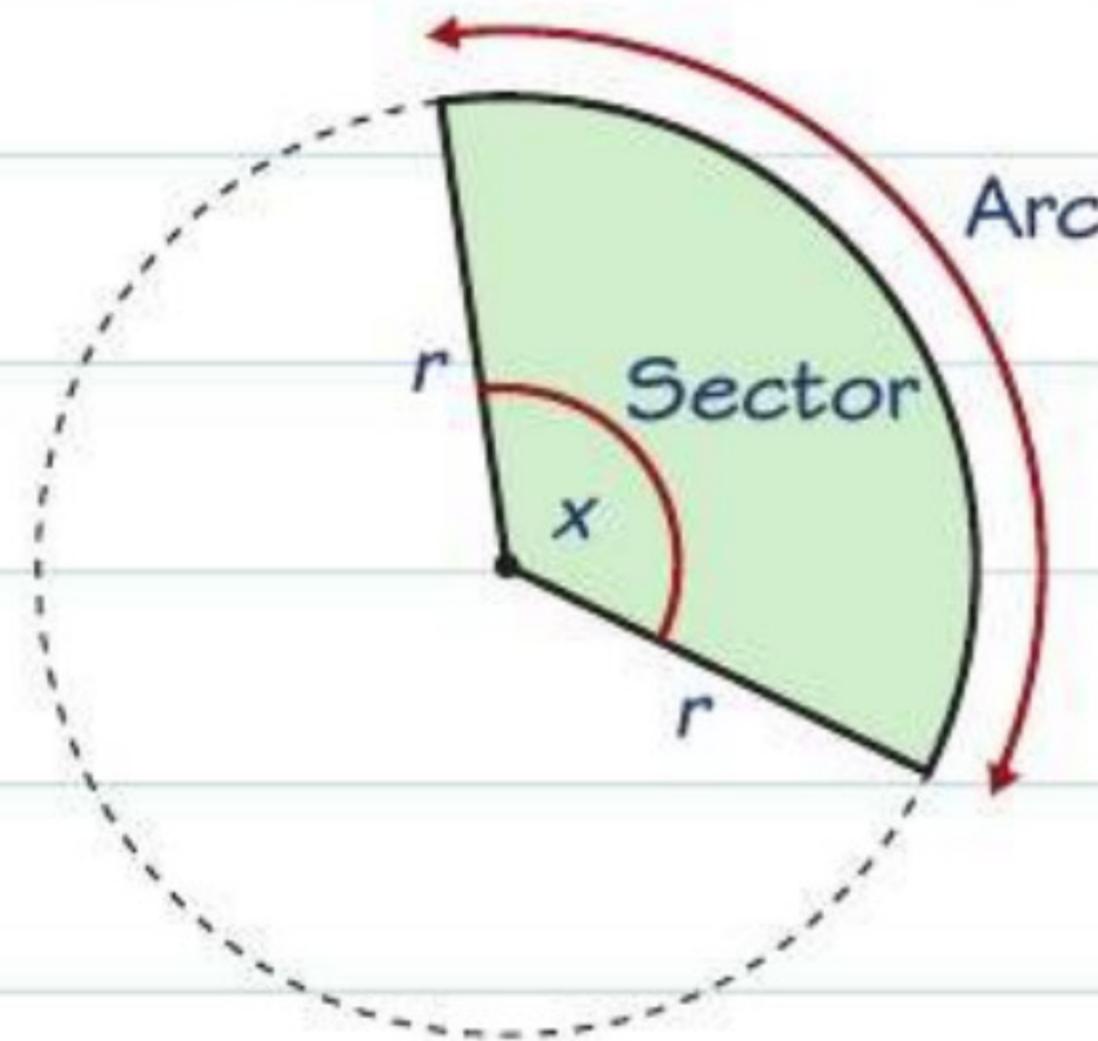
Work out the area of the circle. This is the same as the area of the square. Work out the square root to find x .



Sectors of circles

Each pair of radii divides a circle into two sectors, a **major sector** and a **minor sector**.

You can find the area of a sector by working out what fraction it is of the whole circle.



For a sector with angle x of a circle with radius r :

Sector = $\frac{x}{360^\circ}$ of the whole circle so

Area of sector = $\frac{x}{360^\circ} \times \pi r^2$

Arc length = $\frac{x}{360^\circ} \times 2\pi r$

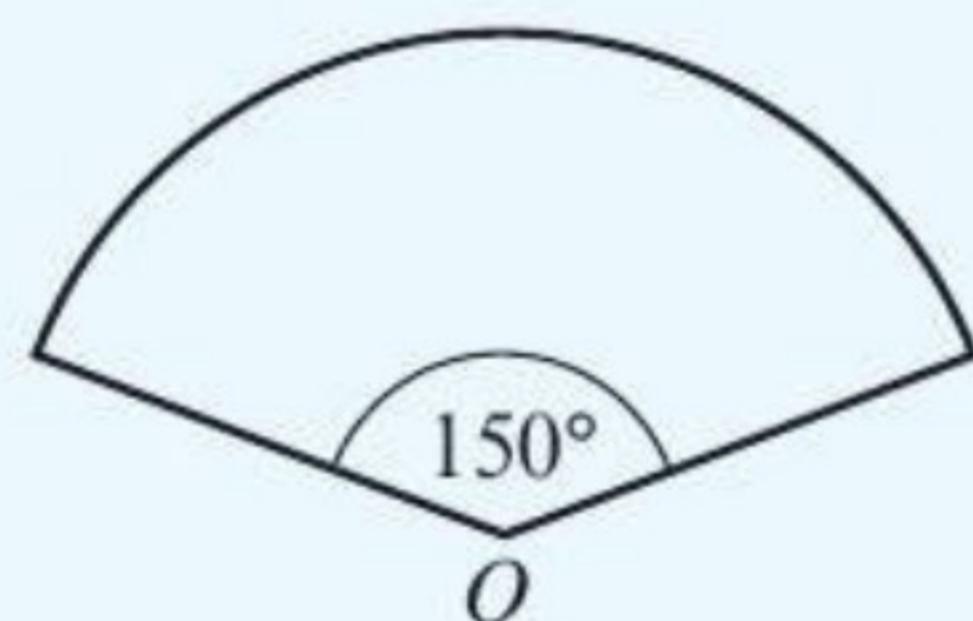
LEARN IT!

You can give answers in terms of π .
There is more about this on the next page.

Worked example

Target grade **5**

The diagram shows a minor sector of a circle of radius 13 cm.



Work out the perimeter of the sector. (4 marks)

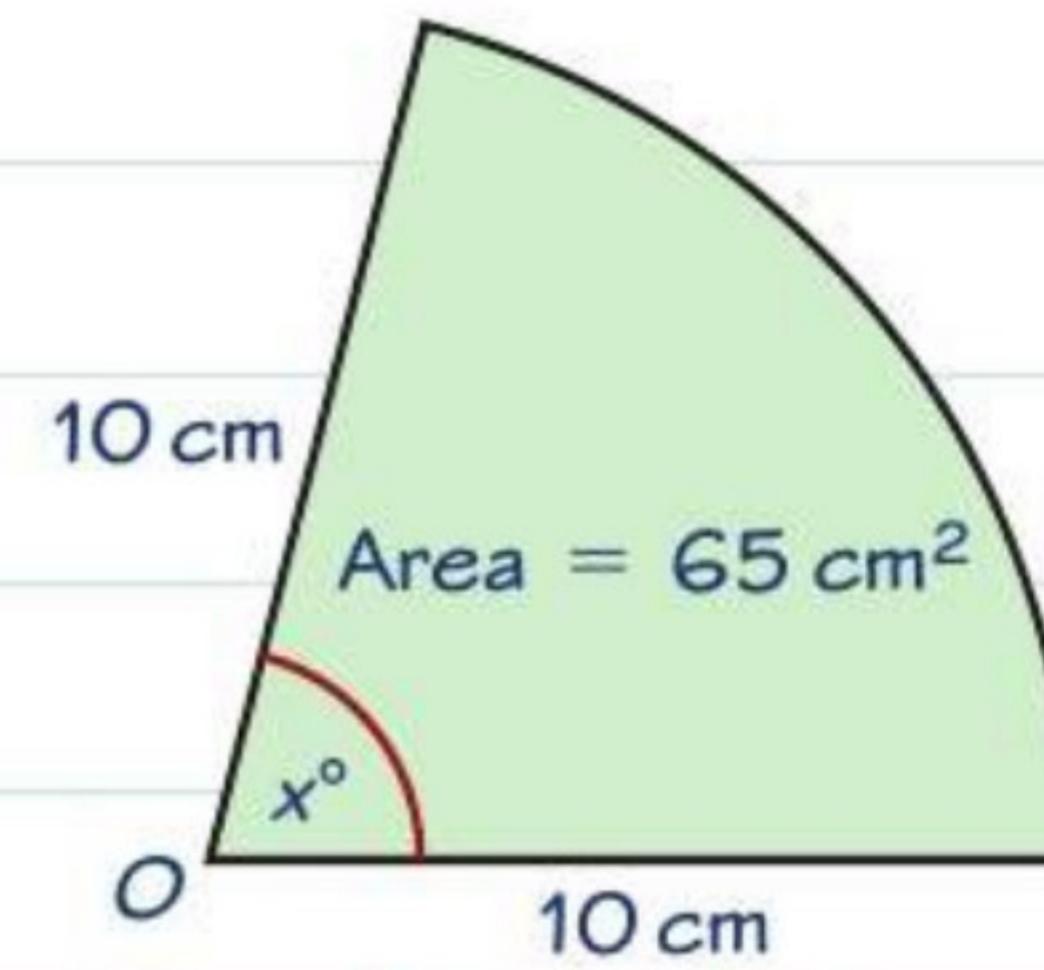
$$\begin{aligned}\text{Arc length} &= \frac{x}{360^\circ} \times 2\pi r \\ &= \frac{150^\circ}{360^\circ} \times 2 \times \pi \times 13 \\ &= 34.03392\dots\end{aligned}$$

$$\begin{aligned}\text{Perimeter} &= \text{Arc length} + \text{Radius} + \text{Radius} \\ &= 34.03392\dots + 13 + 13 \\ &= 60 \text{ cm (2 s.f.)}\end{aligned}$$

Don't round until your final answer. The radius is given correct to 2 significant figures so this is a good degree of accuracy.

Finding a missing angle

You can use the formulae for arc length or area to find a missing angle in a sector. Practise this method to help you tackle the hardest questions.



$$\begin{aligned}\text{Area of sector} &= \frac{x}{360^\circ} \times \pi r^2 \\ 65 &= \frac{x}{360^\circ} \times 100\pi \\ x &= \frac{65 \times 360}{100\pi} \\ &= 74.4845\dots \\ &= 74.5^\circ \text{ (to 3 s.f.)}\end{aligned}$$

Now try this

Target grade **5**

OAB is a sector of a circle, centre O .

Angle $AOB = 60^\circ$.

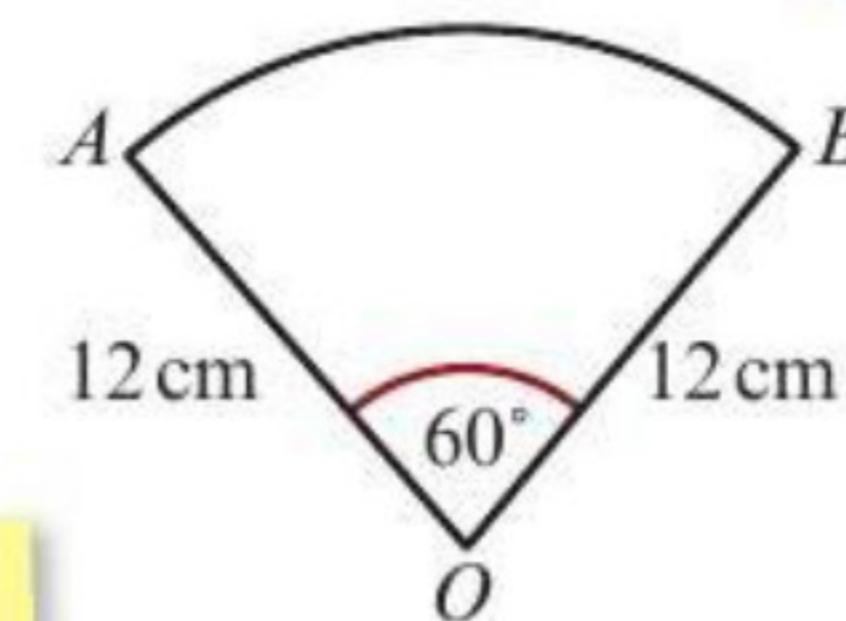
$OA = OB = 12\text{ cm}$.

Work out the length of the arc AB .

Give your answer correct to 3 significant figures.

(3 marks)

You need to learn the formula for arc length.



Cylinders

Surface area

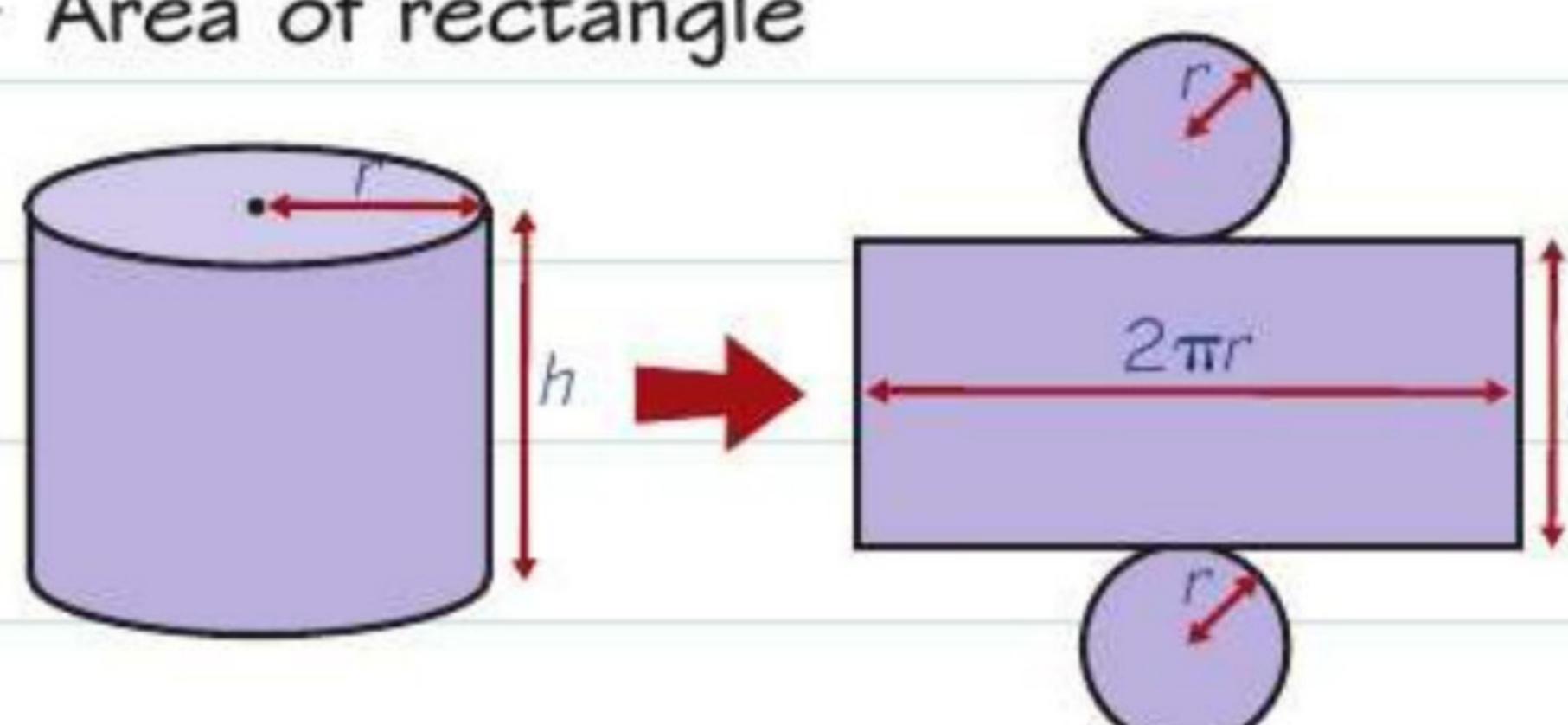
To find the **surface area** of a cylinder you need to add up the areas of the faces. A cylinder has two flat circular faces and one curved face. When you flatten out the curved face it is shaped like a rectangle.

$$\text{Surface area} = 2 \times \text{Area of circle} + \text{Area of rectangle}$$

$$= 2 \times \pi r^2 + 2\pi r \times h$$

$$= 2\pi r^2 + 2\pi r h$$

LEARN IT!



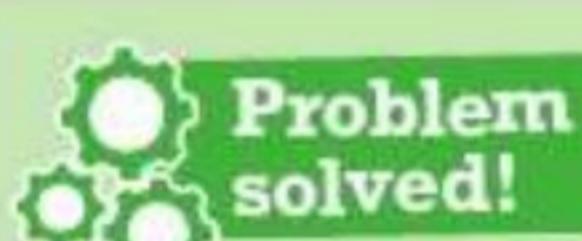
The length of the rectangular face is the same as the circumference of the circle.

Volume

For a cylinder with radius r and height h :

$$\begin{aligned} \text{Volume of cylinder} &= \text{Area of base} \times \text{Height} \\ &= \text{Area of circle} \times \text{Height} \\ &= \pi r^2 h \end{aligned}$$

LEARN IT!



You should always write down the formula

before substituting. Be careful when deciding which quantities to use. You are given the **diameter** but the formula uses **radius** so you need to divide by 2.

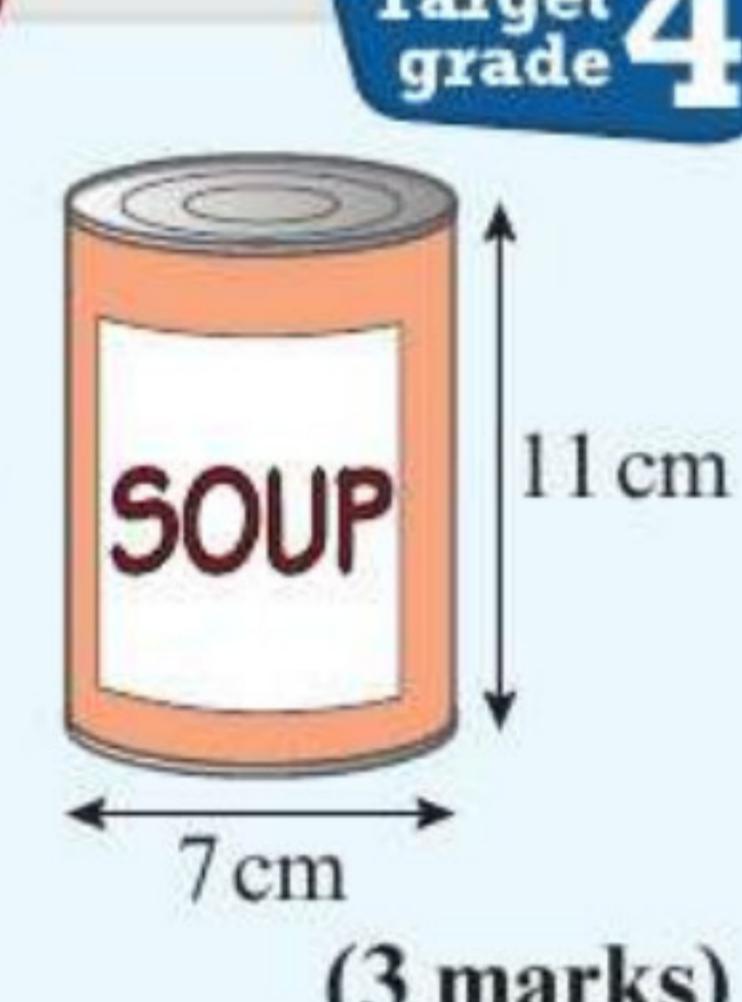
Worked example

Target grade **4**

This tin of soup is in the shape of a cylinder with height 11 cm. The diameter of the base is 7 cm.

$$1 \text{ cm}^3 = 1 \text{ ml}$$

Work out the capacity of the tin in ml.



(3 marks)

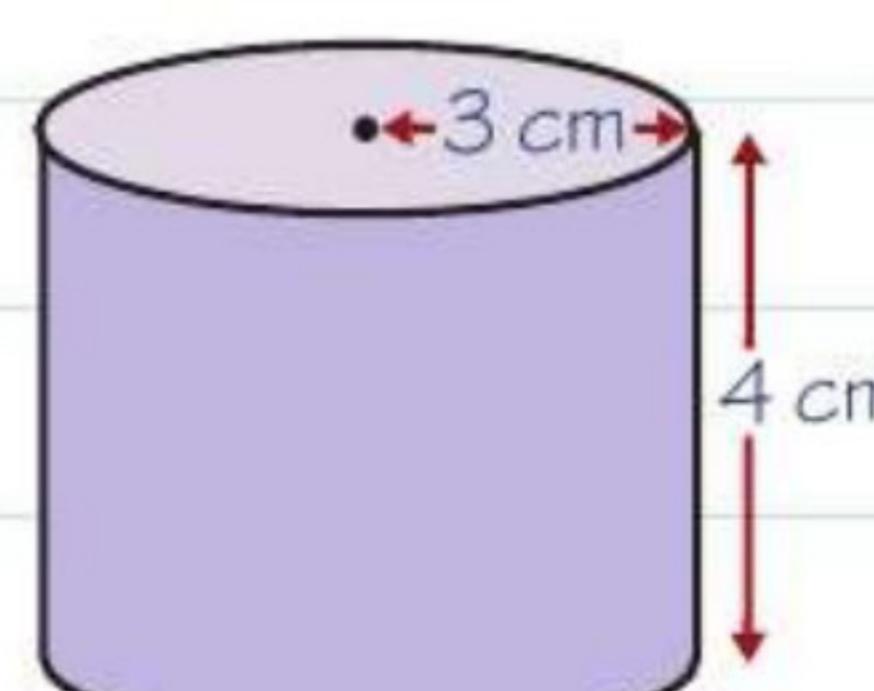
$$\text{Radius of base} = 7 \div 2 = 3.5 \text{ cm}$$

$$\begin{aligned} \text{Volume} &= \pi r^2 h = \pi \times 3.5^2 \times 11 \\ &= 423.3296\ldots \text{ cm}^3 \end{aligned}$$

The capacity is 423 ml to the nearest whole number.

In terms of π

If a question asks for an **exact value** or an answer in **terms of π** then don't use the π button on your calculator. Write your answer as a whole number or fraction multiplied by π .



$$\begin{aligned} \text{Volume of cylinder} &= \pi r^2 h \\ &= \pi \times 3^2 \times 4 \end{aligned}$$

Exact answer

$$\text{Volume} = 36\pi \text{ cm}^3$$

Rounded answer

$$\text{Volume} = 113 \text{ cm}^3 \text{ (to 3 s.f.)}$$

Now try this

Target grade **4**

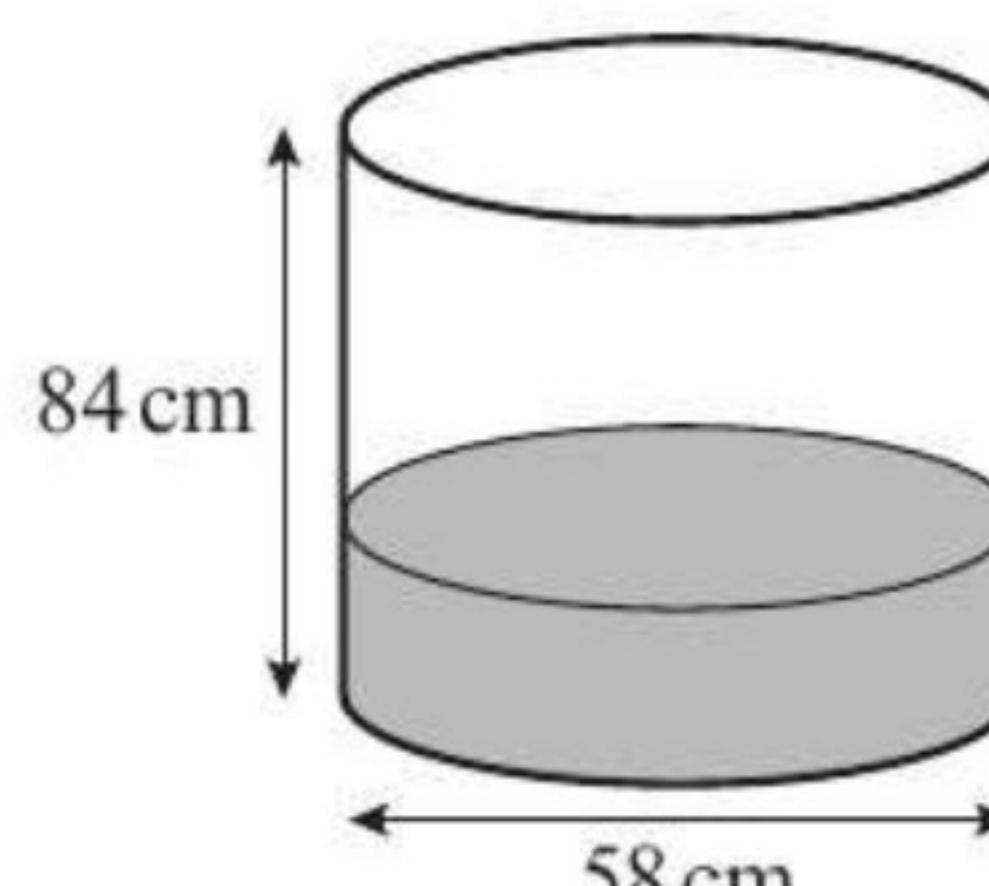
The diagram shows an oil drum in the shape of a cylinder of height 84 cm and diameter 58 cm.

It is one-quarter full of crude oil.

Calculate the volume of oil in the cylinder.

Give your answer in litres, correct to the nearest litre.

(4 marks)



1 litre = 1000 cm³

Volumes of 3-D shapes

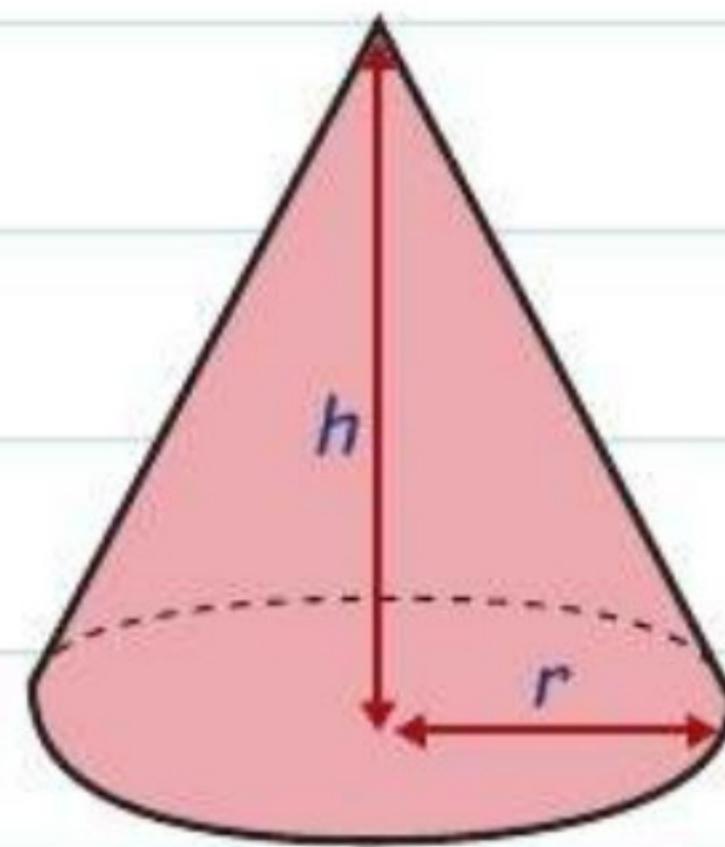
If you need to use these formulae in your exam they will be given to you with the question.

Cone

Volume of cone

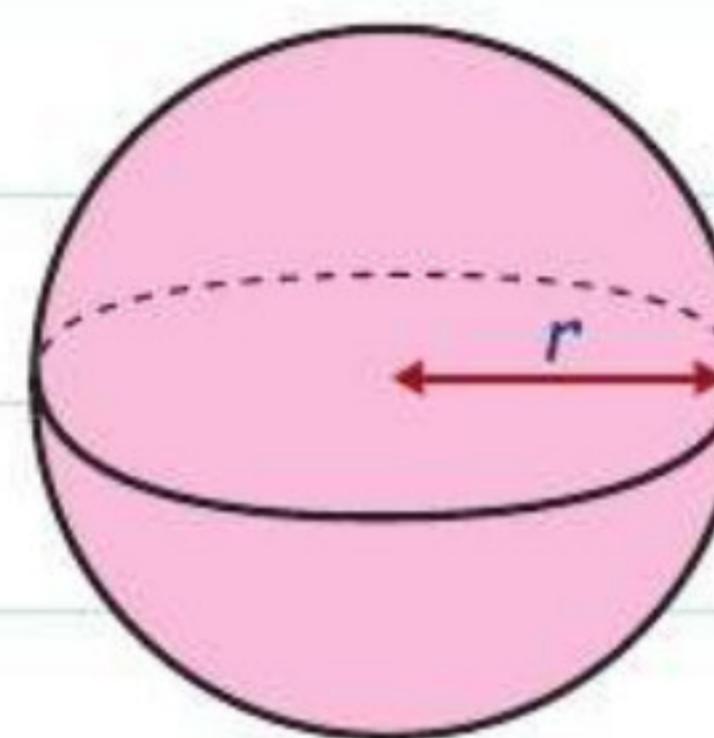
$$= \frac{1}{3} \times \text{Area of base} \times \text{Vertical height}$$

$$= \frac{1}{3} \pi r^2 h$$



Sphere

Volume of sphere = $\frac{4}{3} \pi r^3$

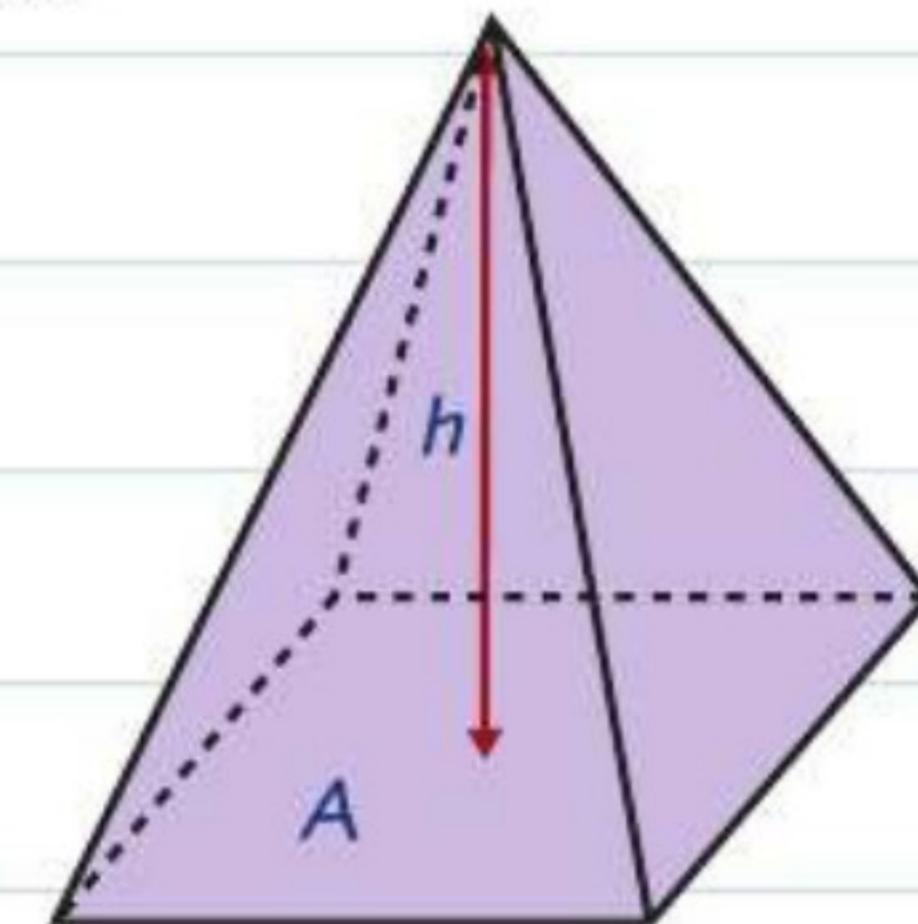


Pyramid

Volume of pyramid

$$= \frac{1}{3} \times \text{Area of base} \times \text{Vertical height}$$

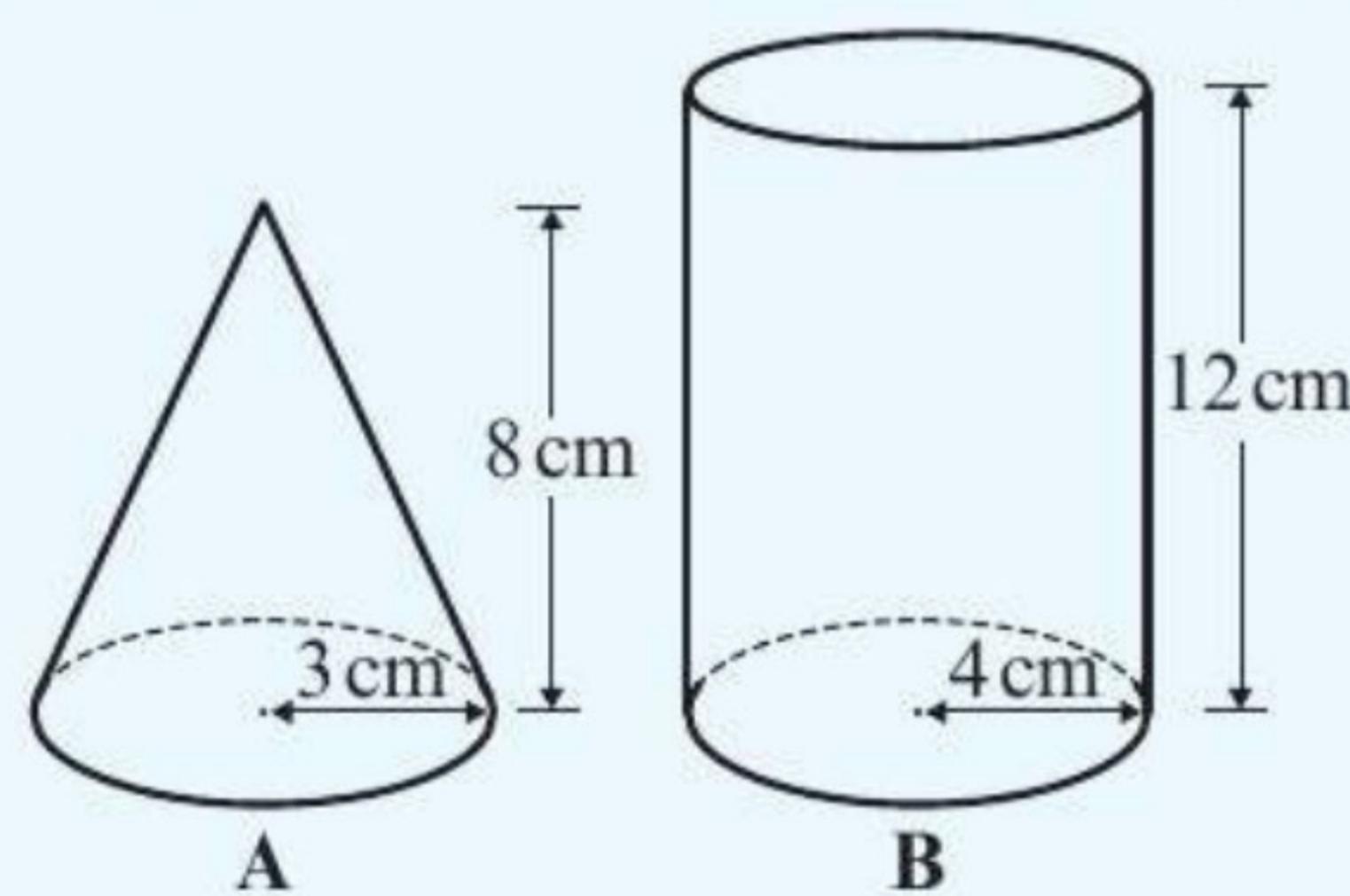
$$= \frac{1}{3} Ah$$



Worked example

Target grade 5

The diagram shows a cone A and a cylinder B.
Show that the volume of B is 8 times the volume of A. (4 marks)



$$\text{Volume of A} = \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi \times 3^2 \times 8 = 24\pi$$

$$\text{Volume of B} = \pi r^2 h = \pi \times 4^2 \times 12 = 192\pi$$

$8 \times 24\pi = 192\pi$ so the volume of B is 8 times the volume of A.

Examiners' report

You might have to **compare** two volumes or areas in your exam. These questions might involve:

- working out the ratio between two different areas or volumes
- finding an unknown quantity represented by a letter
- finding an expression for a length, area or volume in terms of an unknown.

In this question you need to know the ratio between the two volumes. Calculate them both, then write a short **conclusion**. Make sure you show the calculation in your conclusion: $8 \times 24\pi = 192\pi$

You can leave your working in terms of π to make it easier. There is more about this on page 106.

Real students have struggled with questions like this in recent exams – **be prepared!**



Now try this

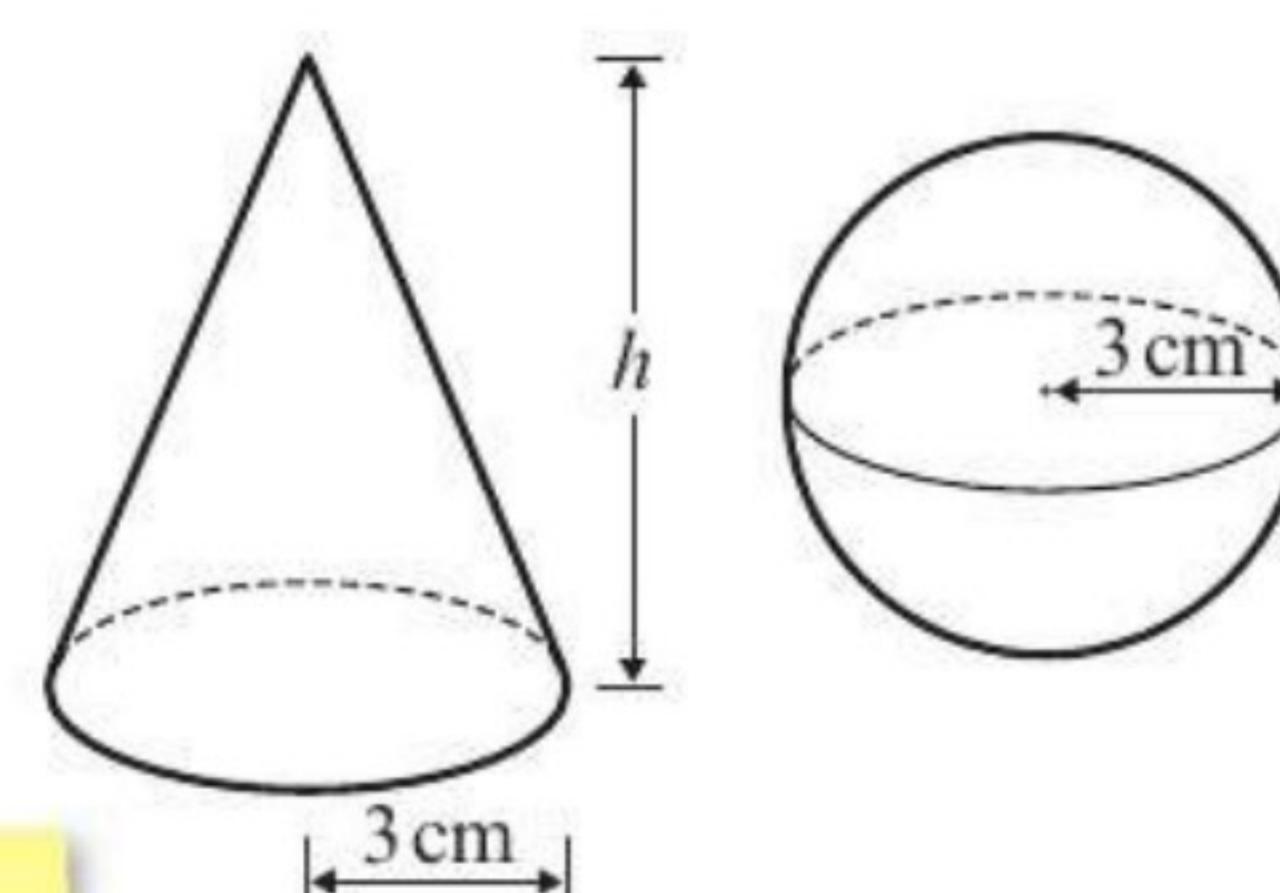
Target grade 5

The radius of the base of a cone is 3 cm and its height is h cm.

The radius of a sphere is 3 cm.

The volume of the cone is equal to the volume of the sphere. Find the value of h . You must show your working. (3 marks)

Write the expression for the volume of each shape; then set them equal to each other. Rearrange to make h the subject.

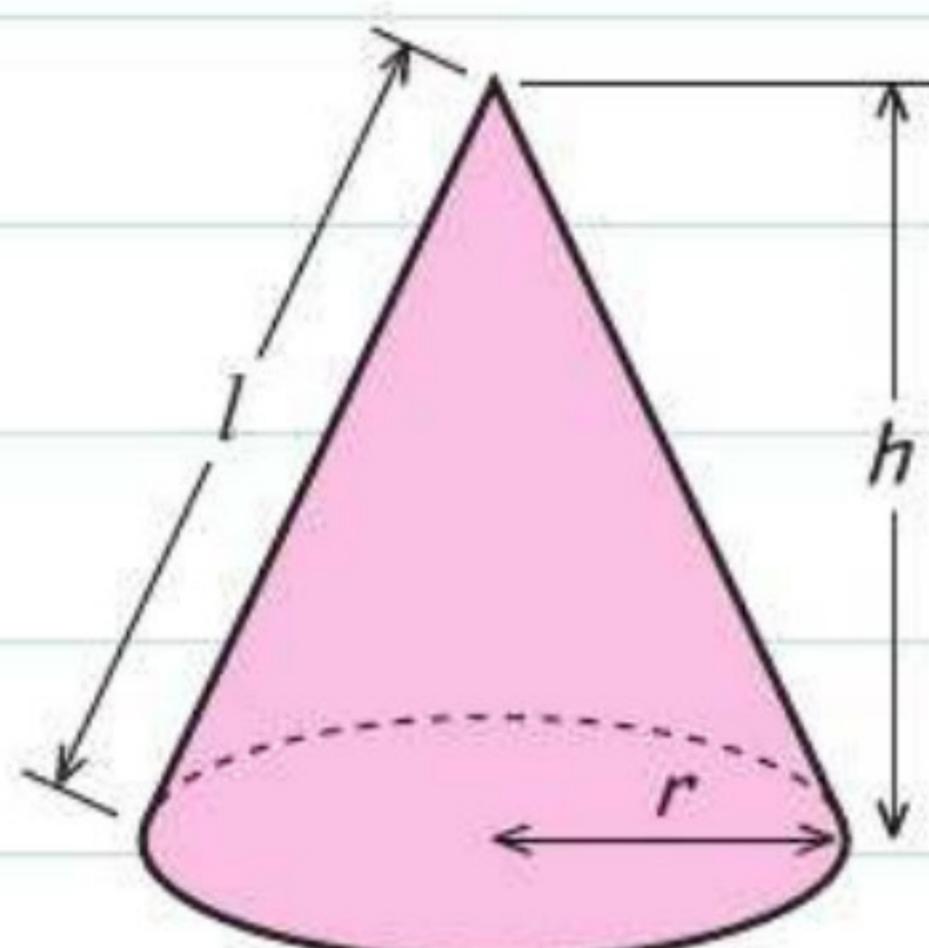


Tricky
Topic

Cone

The formula for the **curved surface area** of a cone will be given if you need it for a question.

$$\text{Curved surface area of cone} = \pi r l$$

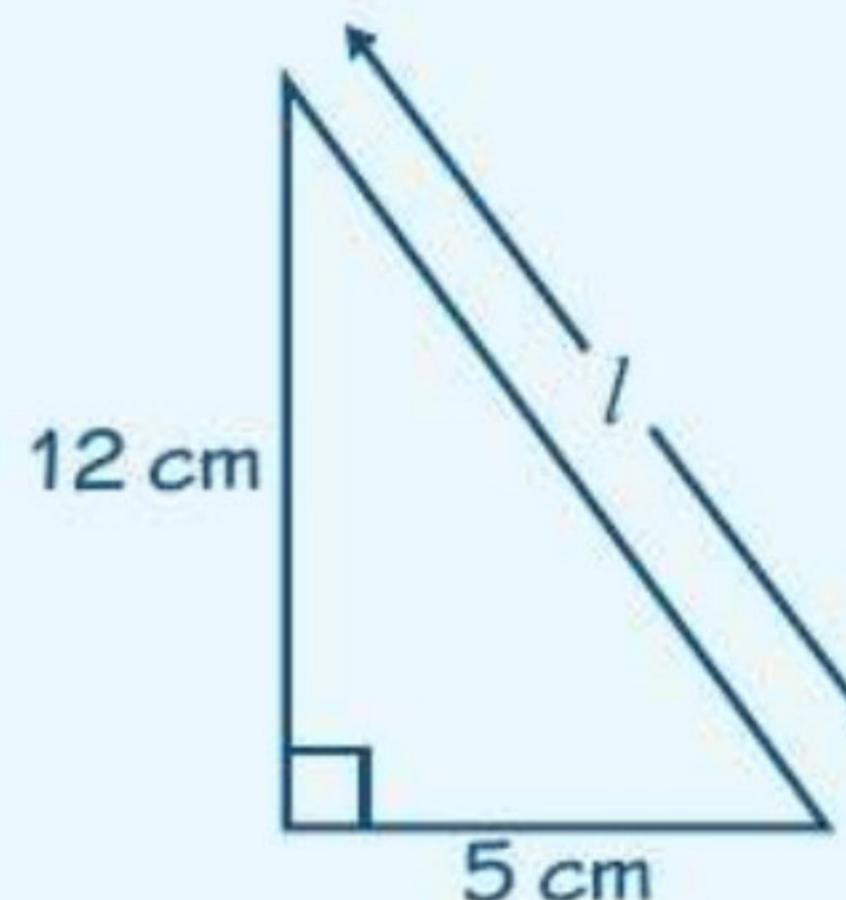


Be careful! This formula uses the slant height, l , of the cone.

To calculate the **total** surface area of the cone you need to add the area of the base. Surface area of cone = $\pi r^2 + \pi r l$

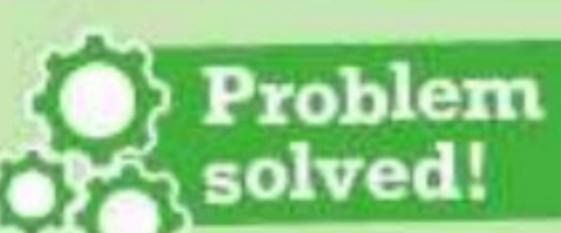
Worked example

The diagram shows a cone with vertical height 12 cm and base radius 5 cm. Work out the curved surface area of the cone. (4 marks)



$$\begin{aligned} l^2 &= 12^2 + 5^2 = 169 \\ l &= 13 \text{ cm} \\ \text{Curved surface area} &= \pi r l \\ &= \pi \times 13 \times 5 \\ &= 65\pi \text{ cm}^2 \end{aligned}$$

Target grade 5



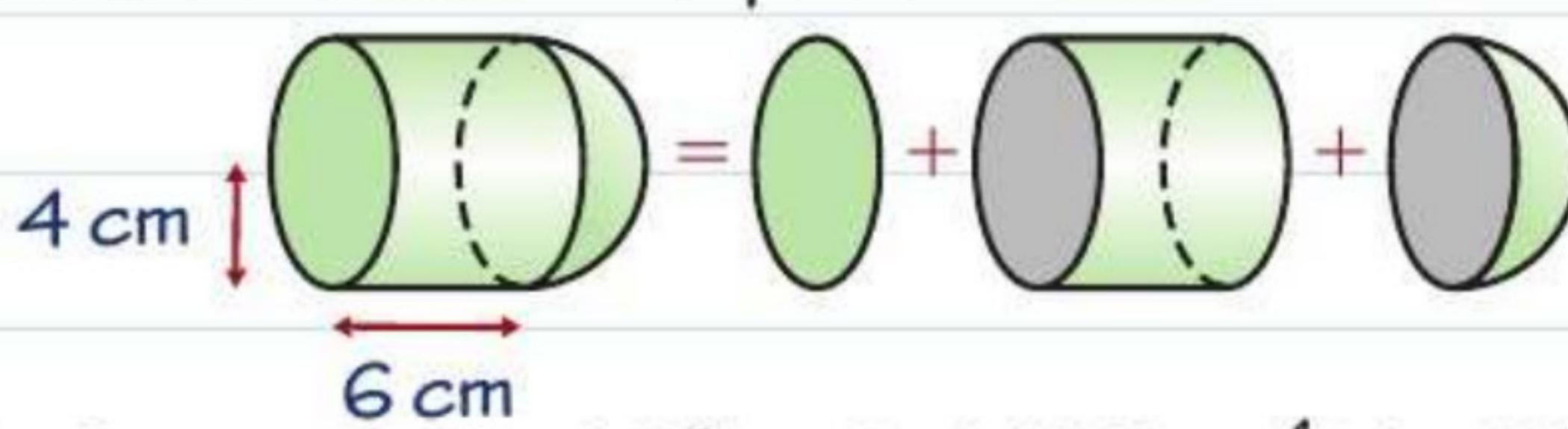
To work out the curved surface area you need to know the radius and the slant height. You are given the radius and the **vertical height**.

To calculate the slant height you need to use Pythagoras' theorem. Sketch the right-angled triangle containing the missing length.



Compound shapes

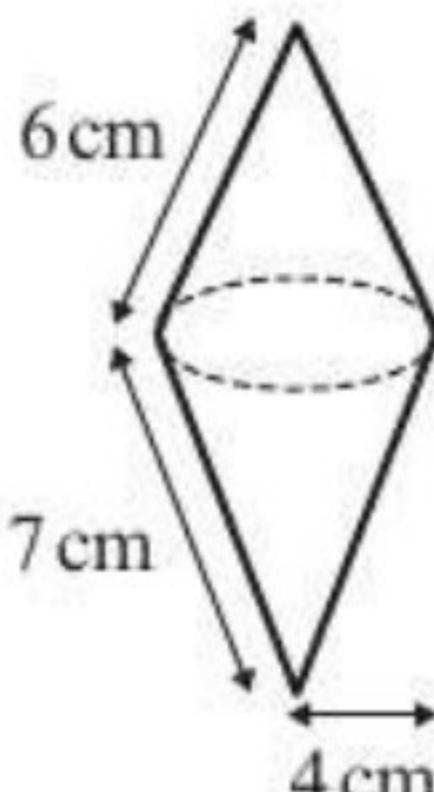
You can calculate the surface area of more complicated shapes by adding together the surface area of each part.



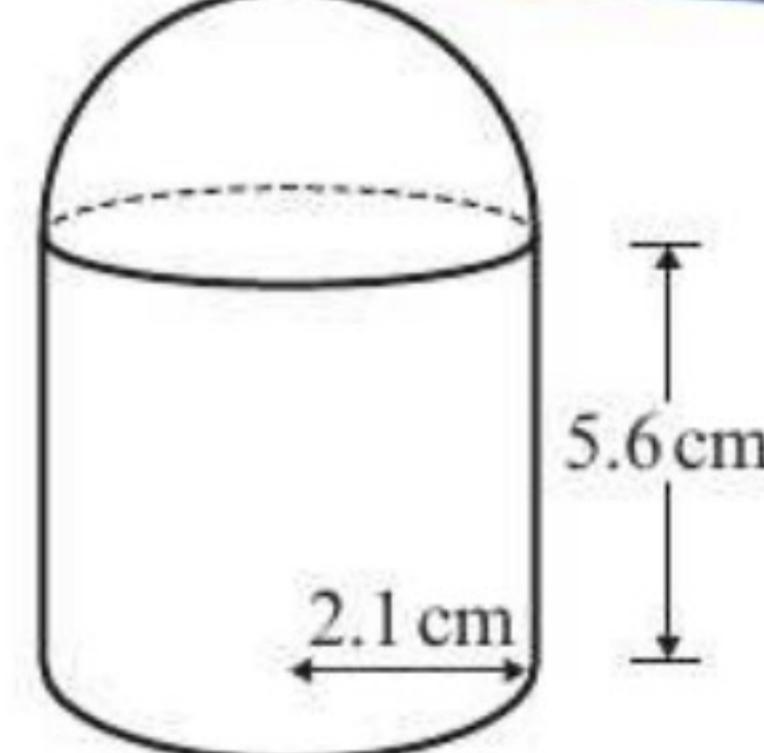
$$\begin{aligned} \text{Surface area} &= \pi(4)^2 + 2\pi(4)(6) + \frac{1}{2}[4\pi(4)^2] \\ &= 96\pi \text{ cm}^2 \end{aligned}$$

Now try this

1 The diagram shows an object made from two cones, placed one on top of the other. The top cone has a slant height of 6 cm and the bottom cone has a slant height of 7 cm. Both cones have a radius of 4 cm. Work out the total surface area of the object. Give your answer in terms of π . (4 marks)



2 A solid object is formed by joining a hemisphere to a cylinder. Both the hemisphere and the cylinder have a radius of 2.1 cm. The cylinder has a height of 5.6 cm. Work out the total surface area of the object. Give your answer to 3 significant figures. (4 marks)



Target grade 5