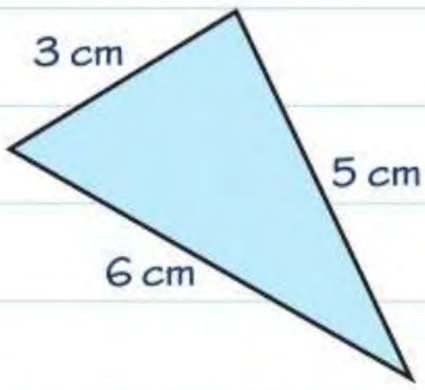
Had a look ☐ Nearly there ☐ Nailed it! ☐

Perimeter and area

Perimeter

Perimeter is the distance around the edge of a shape. You can work out the perimeter of a shape by adding up the lengths of the sides.

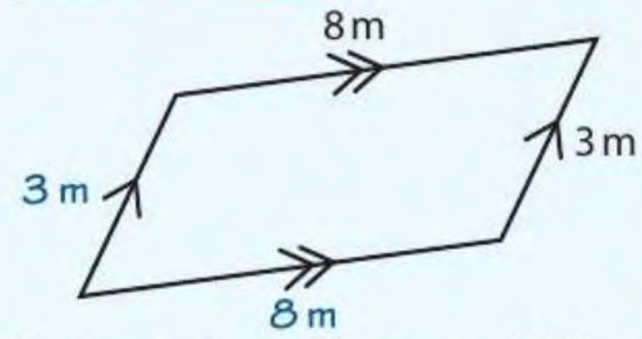


Perimeter = 3 cm + 5 cm + 6 cm
= 14 cm

You might need to measure a shape to find the perimeter.

See page 96 for help on measuring lines.

Worked example Target grade 2



Everything in blue is part of the answer.

Work out the perimeter of this parallelogram. **(2 marks)**

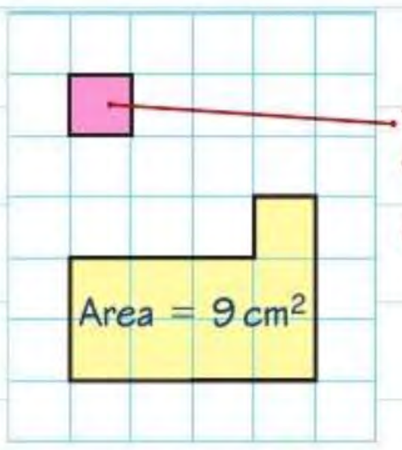
3 + 8 + 3 + 8 = 22

Perimeter = 22 m

Work out the missing lengths first. The opposite sides of a parallelogram are equal so you can fill in these lengths on the diagram.

Area

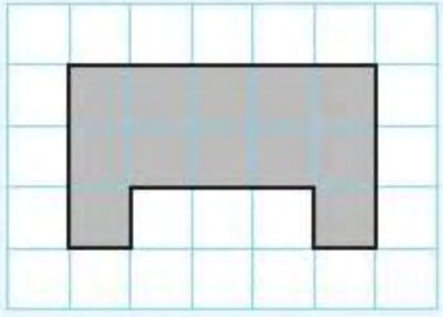
You can work out the **area** of a shape drawn on squared paper by counting the squares.



This area is 1 cm²
You say 'one centimetre squared' or 'one square centimetre'.

Worked example Target grade 1

This shape is drawn on cm squared paper.



(a) Work out the perimeter of the shape. **(1 mark)**

18 cm

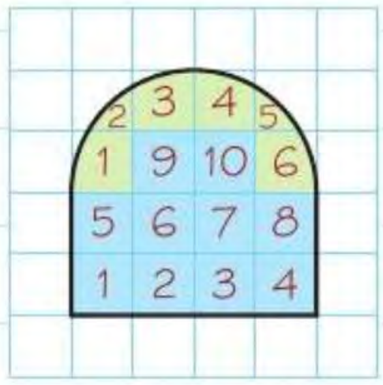
(b) Work out the area of the shape. **(1 mark)**

12 cm²

Estimating

You might need to estimate the area of a shape drawn on cm squared paper. Count 1 cm² for every whole square and $\frac{1}{2}$ cm² for every part square.

Here there are 10 whole squares and 6 part squares. A good estimate is 13 cm².

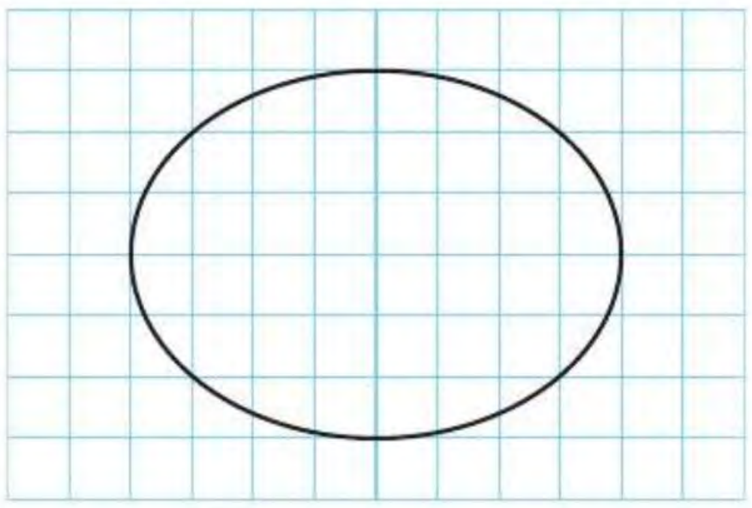


Now try this

Target grade 1

Estimate the area of this oval shape. Each square represents 1 cm². **(2 marks)**

Count 1 cm² for each whole square and $\frac{1}{2}$ cm² for each part square.



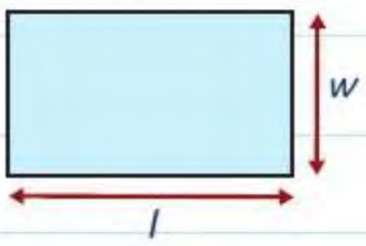
Had a look ☐ Nearly there ☐ Nailed it! ☐

Hot Topic

Area formulae

You will not be given any of the formulae on this page in your exam. Make sure you **learn** them and know how to use them.

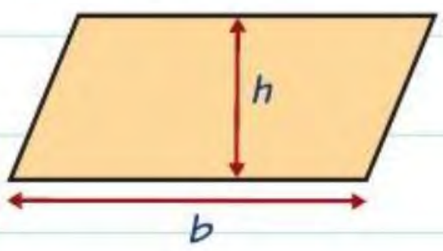
Rectangle



Area = Length × Width

$A = lw$

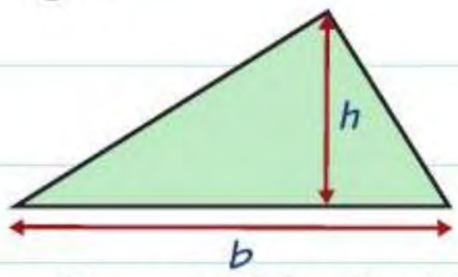
Parallelogram



Area = Base × Vertical height

$A = bh$

Triangle

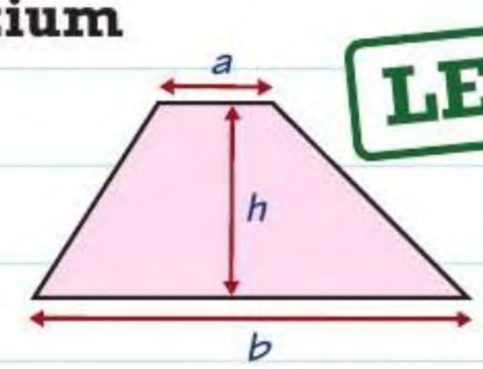


$$\text{Area} = \frac{\text{Base} \times \text{Vertical height}}{2}$$

$A = \frac{1}{2}bh$

h is the vertical height.

Trapezium



LEARN IT!

$$\text{Area} = \frac{1}{2}\left(\begin{matrix}\text{Sum of the two}\\ \text{parallel sides}\end{matrix}\right) \times \text{Vertical height}$$

$A = \frac{1}{2}(a + b)h$

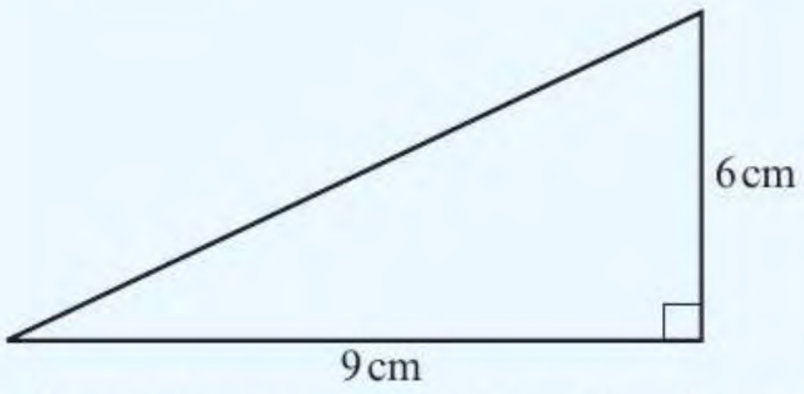
Area checklist

- Make sure lengths are all in the same units.
- Remember to give units with your answer.

Lengths in cm then area in cm^2.
Lengths in m then area in m^2.

Worked example

Target grade 3



Work out the area of this triangle. **(2 marks)**

$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 9 \times 6$

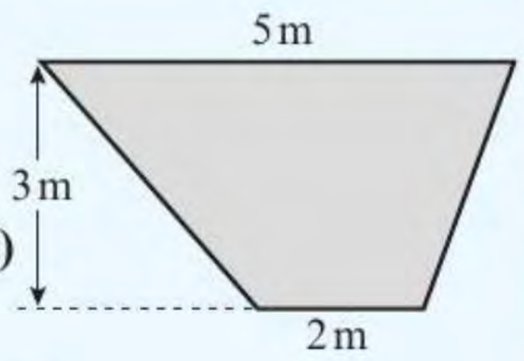
$= 27\ cm^2$

Multiplying by $\frac{1}{2}$ is the same as dividing by 2.

Worked example

Target grade 3

The diagram shows a rug in the shape of a trapezium.
Work out the area of the rug. **(3 marks)**



$A = \frac{1}{2}(a + b)h$

$= \frac{1}{2} \times (2 + 5) \times 3$

$= \frac{1}{2} \times 7 \times 3$

$= 10.5\ m^2$

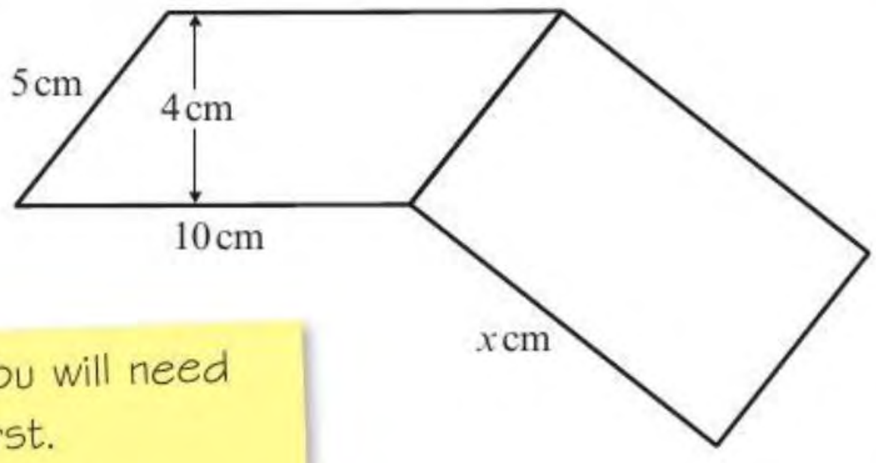
Learn the formula for the area of a trapezium.
You should always **write down** the formula before you substitute any values.

Worked solution video

Now try this

Target grade 3

Here is a diagram of a parallelogram and a rectangle.
The parallelogram has the same area as the rectangle.
Work out the length *x*. **(4 marks)**

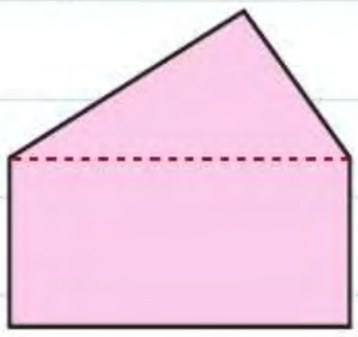


Plan your answer before you start writing. You will need to work out the area of the parallelogram first.
You can write any lengths you find on your diagram.

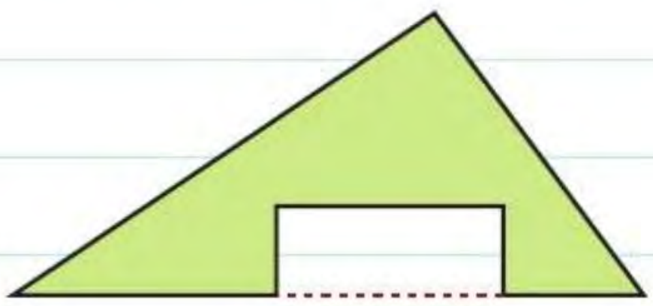
Had a look ☐ Nearly there ☐ Nailed it! ☐

Solving area problems

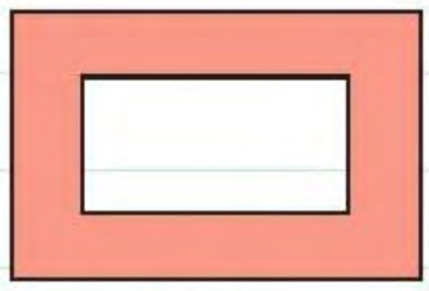
You can calculate areas and perimeters of harder shapes by splitting them into parts.
You might need to draw some extra lines on your diagram and add or subtract areas.



Area = Rectangle + Triangle



Area = Triangle − Rectangle

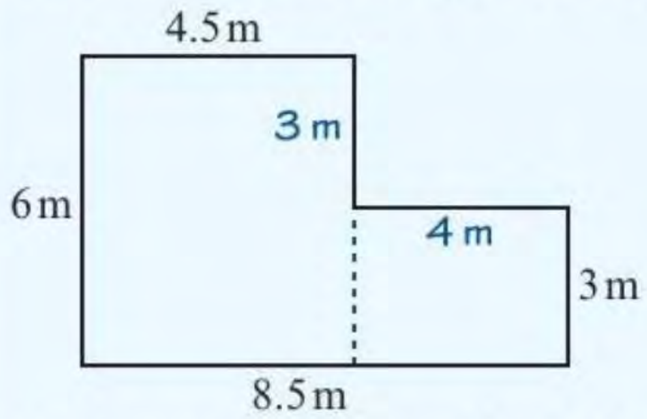


Area = Large rectangle − Small rectangle

Worked example

Target grade 3

The diagram shows a garden bed.



Adrian wants to cover the bed with grass seed.
A packet of grass seed will cover 10 m².

(a) How many packets of grass seed does Adrian need to buy? **(3 marks)**

Area = 6 × 4.5 + 4 × 3
= 27 + 12
= 39 m²

Adrian needs to buy 4 packets of grass seed.

Adrian also wants to build a fence around the edge of the garden bed.

(b) Calculate the total length of Adrian's fence. **(1 mark)**

6 + 4.5 + 3 + 4 + 3 + 8.5 = 29 m

Everything in blue is part of the answer.

Examiners' report

Read the question carefully to decide whether you need to find an **area** or a **perimeter**. Start by finding missing lengths and **write them on the diagram**.

Real students have struggled with questions like this in recent exams – **be prepared!**

Draw a dotted line to divide the garden bed into two rectangles.

You have to use the information in the question to work out the missing lengths.

8.5 m − 4.5 m = 4 m

6 m − 3 m = 3 m

Make sure you answer the question that has been asked.

For part (a) you need to say how many packets of grass seed Adrian needs to buy.

Don't measure the distances. Unless you are told otherwise, all the diagrams in your exam are not drawn accurately.

Now try this

Target grade 3

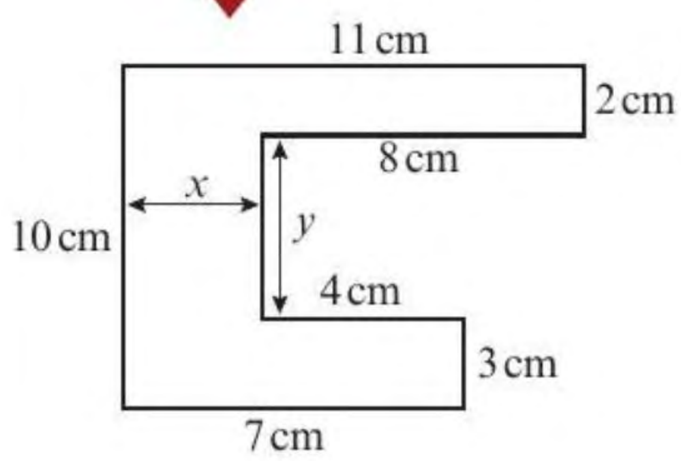
Here is a shape made up of rectangles.

(a) Work out the distance marked x on the diagram. **(1 mark)**

(b) Work out the distance marked y on the diagram. **(1 mark)**

(c) Work out the area of this shape. **(3 marks)**

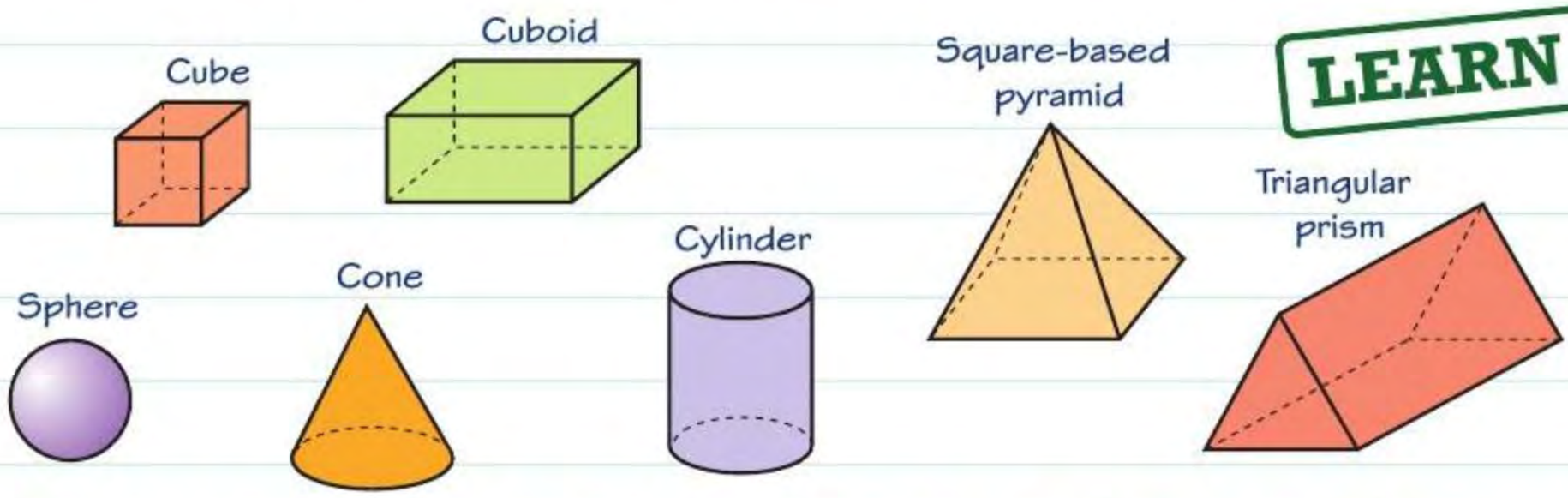
(d) Work out the perimeter of this shape. **(2 marks)**



Had a look ☐ Nearly there ☐ Nailed it! ☐

3-D shapes

You need to learn the names of these 3-D shapes.

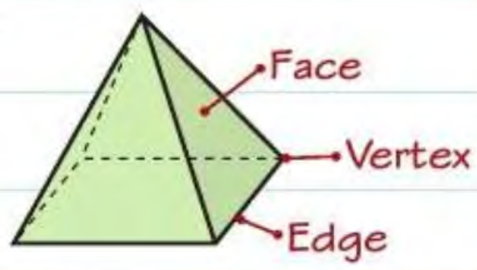


LEARN IT!

Faces, edges and vertices

This square-based pyramid has 5 faces, 8 edges and 5 vertices.

The plural of vertex is vertices.

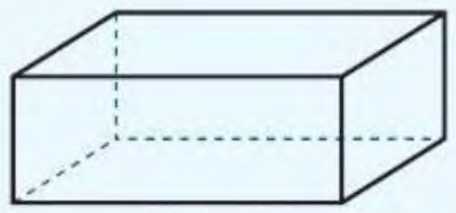


You need to **learn** the mathematical names of 3-D solids. You can sketch the hidden edges to help you work out the answer to part (b).

Worked example Target grade 1

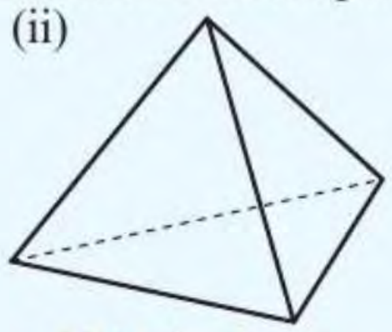
(a) Write down the name of each 3-D shape.

(i)



Cuboid

(ii)



Tetrahedron

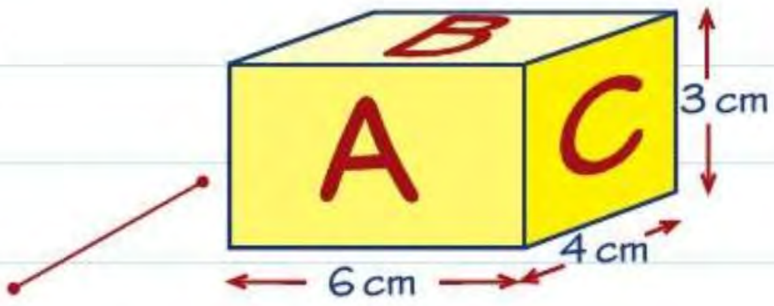
(b) Write down the number of edges on shape (i).

12

Everything in blue is part of the answer.

Surface area of a cuboid

You can find the surface area of a 3-D shape by adding together the areas of its faces. A cuboid has **six** faces. You can use the fact that ***opposite faces*** are ***equal*** to simplify your working.



Surface area $= 2A + 2B + 2C$

$= 2 \times (6 \times 3) + 2 \times (6 \times 4) + 2 \times (4 \times 3)$

$= 2 \times 18 + 2 \times 24 + 2 \times 12 = 108\,cm^2$

Now try this

Worked solution video

Target grade 3

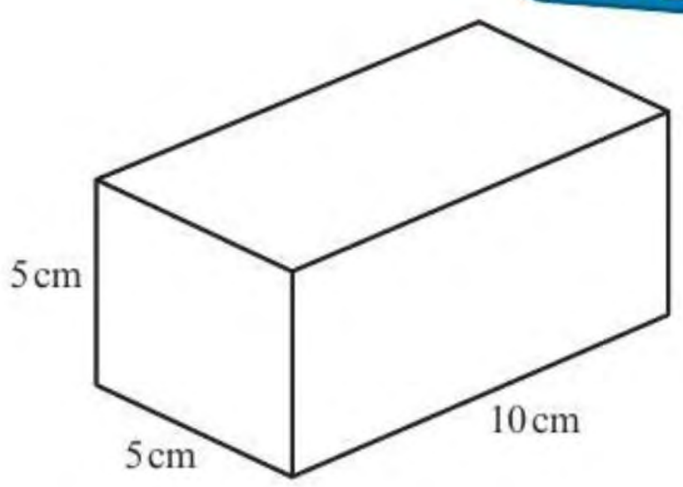
The diagram shows a solid block of wood in the shape of a cuboid.

(a) Work out the total surface area of the cuboid. **(2 marks)**

Karl has to paint all 6 faces of 50 cuboids.

A can of spray paint covers an area of 3000 cm^2.

(b) How many cans of paint must Karl buy to paint all the faces? Show all your working. **(3 marks)**

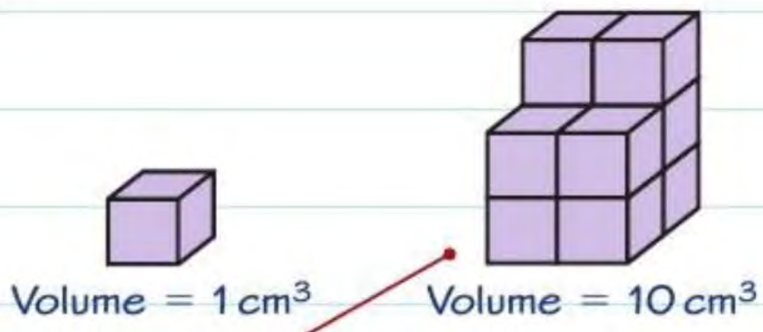


Don't just write down a number or estimate. You must show all your working.

Volumes of cuboids

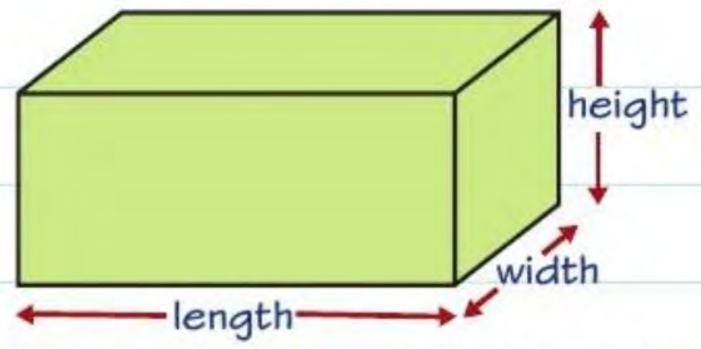
The **volume** of a 3-D shape is the amount of space it takes up.

The most common units of volume are cm^3 or m^3.



Volume of a cuboid

You need to remember this formula for the volume of a **cuboid**.

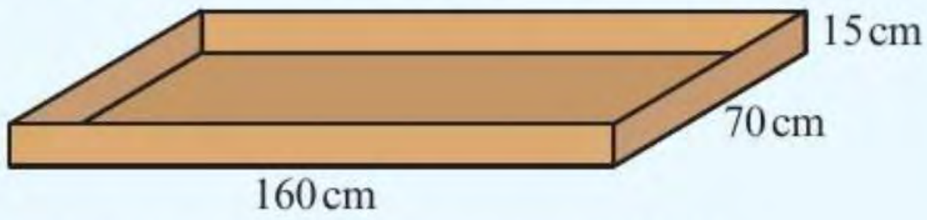


Volume = Length × Width × Height

Worked example

Target grade 3

The diagram shows a wooden planting box.



A 50 litre bag of compost costs £3.99.
1 litre = 1000 cm^3
How much will it cost to fill this planting box?
(4 marks)

Volume = 160 × 70 × 15
= 168 000 cm^3
= 168 litres
4 bags of compost are needed.
4 × £3.99 = £15.96

Problem solved!

There are a few steps to this problem so it's a good idea to plan your answer:

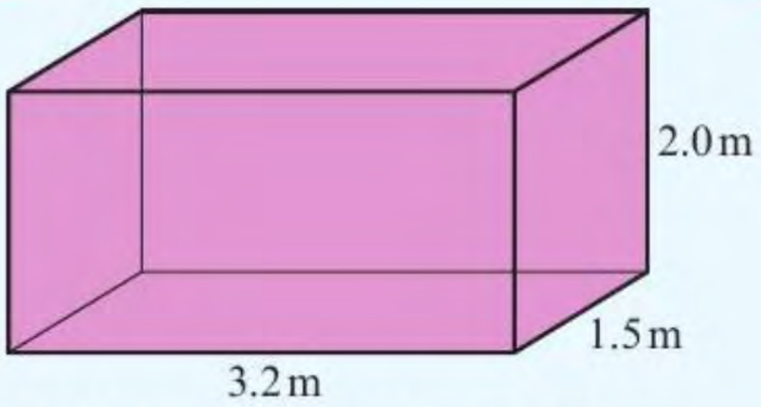
1. Work out the volume of the planting box.
2. Convert from cm^3 to litres by dividing by 1000.
3. Work out how many bags of compost are needed.
4. Work out how much the compost will cost.

Remember that you can only buy a **whole number** of bags of compost.

3 × 50 litres = 150 litres
4 × 50 litres = 200 litres
You need 168 litres of compost, so you need to buy 4 bags.

Worked example

Target grade 2



Work out the volume of this cuboid.
(2 marks)

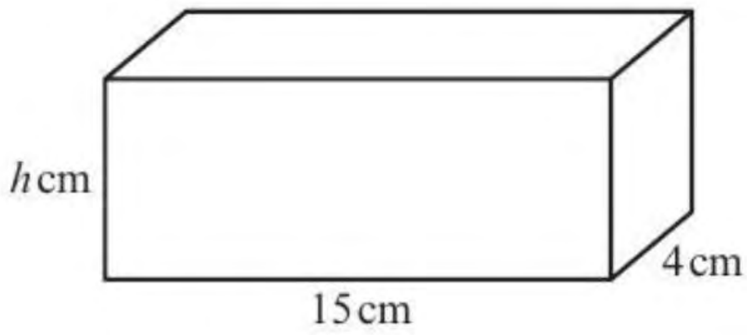
Volume = Length × Width × Height
= 3.2 × 1.5 × 2.0
= 9.6 m^3

The units of length are m so the units of volume will be m^3.

Now try this

Target grade 3

This cuboid has a volume of 390 cm^2.



Work out the height, h cm, of the cuboid.
(3 marks)

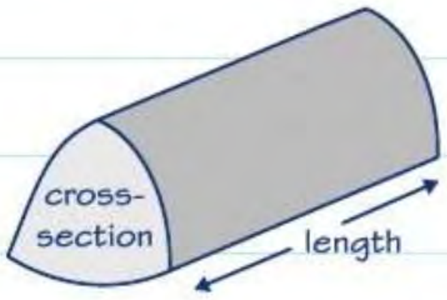
15 × 4 × h = 390

Had a look ☐ Nearly there ☐ Nailed it! ☐

Prisms

Volume

A prism is a 3-D solid with a **constant cross-section**. Use this formula to calculate the volume of a prism.



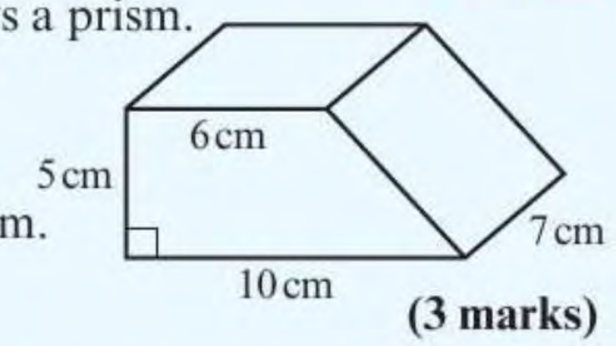
Volume = Area of cross-section × Length

LEARN IT!

Worked example

Target grade 4

The diagram shows a prism. The cross-section is a trapezium. Work out the volume of the prism.



(3 marks)

Area of cross-section (trapezium)

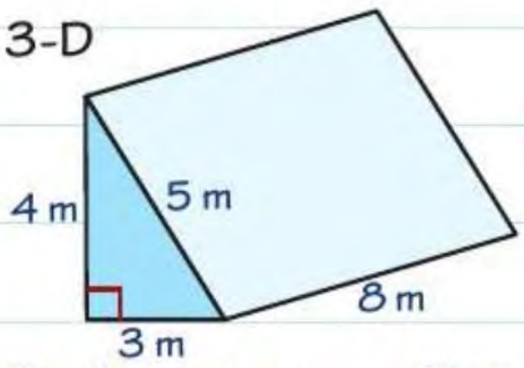
$= \frac{1}{2} \times (6 + 10) \times 5 = 40\,cm^2$

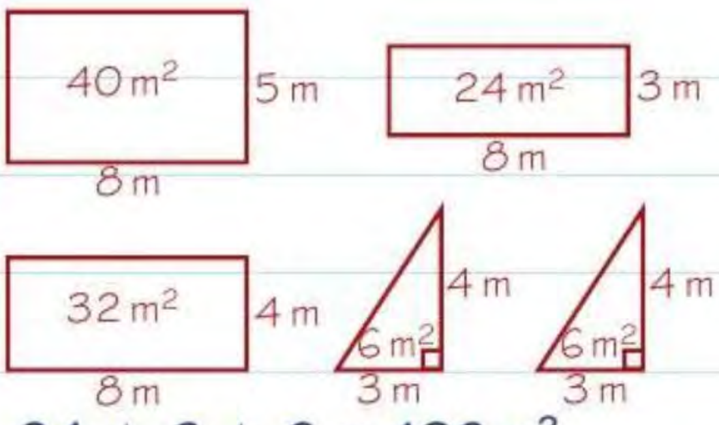
Volume of prism $= 40 \times 7 = 280\,cm^3$

Surface area

To work out the surface area of a 3-D shape you need to add together the areas of all the faces.

It's a good idea to sketch each face with its dimensions. Remember to include the faces that you can't see.



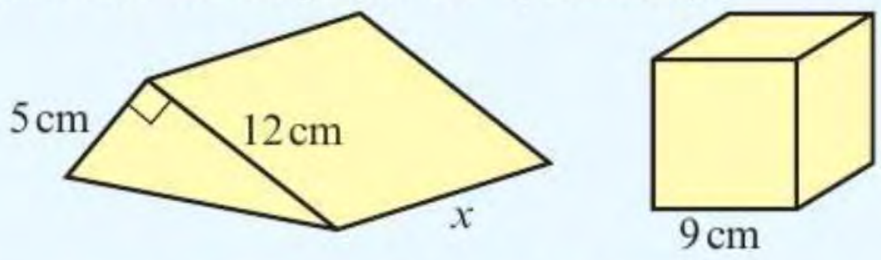


Surface area $= 40 + 32 + 24 + 6 + 6 = 108\,m^2$

Worked example

Target grade 4

The diagram shows a triangular prism and a cube. They both have the **same** volume. Work out the length of x. **(4 marks)**



Volume of cube $= 9^3 = 729\,cm^3$

Volume of prism = Area of cross-section × Length

$= \frac{1}{2} \times 5 \times 12 \times x = 30x$

$30x = 729$

$x = 24.3$ cm

Problem solved! Calculate the volume of the cube, and write an expression for the volume of the prism. Set these equal to each other and solve the equation to find x.

Examiners' report

Write down what you are calculating at each stage. You can get marks for your working even if your final answer is wrong.

Real students have struggled with questions like this in recent exams – **be prepared!**

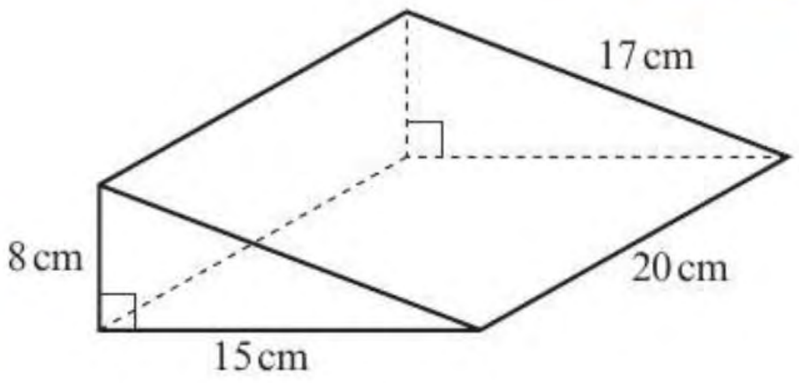
Now try this

Target grade 4

The cross-section of this prism is a right-angled triangle.

(a) Work out the volume of the prism. **(3 marks)**

(b) Work out the total surface area of the prism. **(3 marks)**



Use this formula to work out the areas of the triangular faces:

Area of a triangle $= \frac{1}{2} \times$ Base × Vertical height.

Had a look ☐ Nearly there ☐ Nailed it! ☐

Hot Topic

Area of a circle

You need to know the formula for the area of a circle:

Area $= \pi \times \text{Radius}^2$

$A = \pi \times r \times r = \pi r^2$

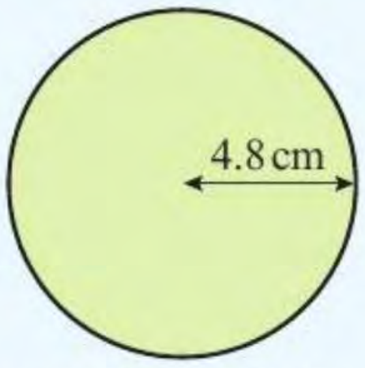
LEARN IT!

You always need to use the **radius** when you are calculating the area.

If you are given the diameter, divide it by 2 to get the radius.

Worked example

Target grade 3



Work out the area of this circle. **(2 marks)**

Area $= \pi r^2$

$= \pi \times 4.8^2$

$= \pi \times 23.04$

$= 72.3822...$

$= 72.4\,\text{cm}^2$ (1 d.p.)

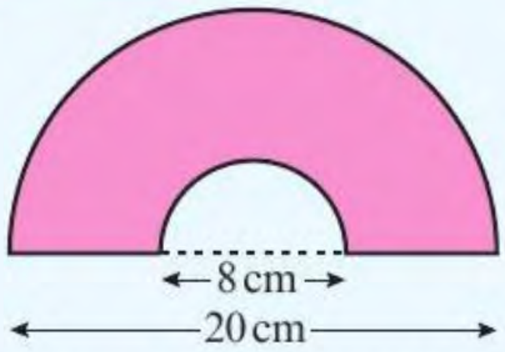
Examiners' report

When you have to keep track of a lot of working, **write down** what each calculation represents. It will be really clear to you **and** the examiner what you have worked out!

Real students have struggled with questions like this in recent exams – **be prepared!**

Worked example

Target grade 4



The diagram shows two semicircles with diameters 8 cm and 20 cm.
Work out the shaded area. **(4 marks)**

Radius of large semicircle $= 20 \div 2$

$= 10$

Area of large semicircle $= \frac{1}{2} \times \pi \times 10^2$

$= 157.0796...$

Radius of small semicircle $= 8 \div 2 = 4$

Area of small semicircle $= \frac{1}{2} \times \pi \times 4^2$

$= 25.1327...$

Shaded area $= 157.0796... - 25.1327...$

$= 131.946...$

$= 132\,\text{cm}^2$ (3 s.f.)

Examiners' report

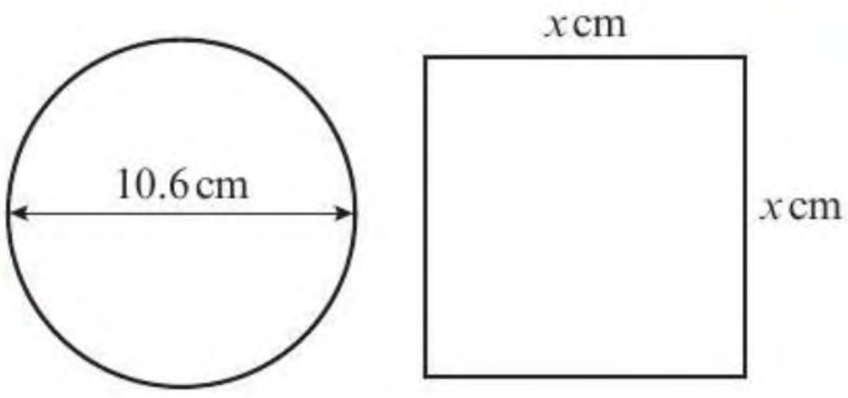
The formula for the area of a circle uses the **radius**. If you are given the **diameter** you will need to divide by 2 first.

Real students have struggled with questions like this in recent exams – **be prepared!**

Now try this

Target grade 4

This circle and this square are equal in area.
Work out the length of the side of the square.
Give your answer correct to 1 decimal place.
(4 marks)



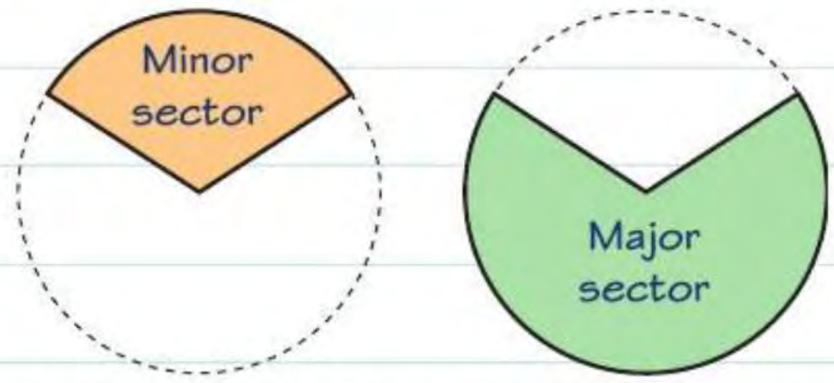
Work out the area of the circle. This is the same as the area of the square. Work out the square root to find x.

Had a look ☐ Nearly there ☐ Nailed it! ☐

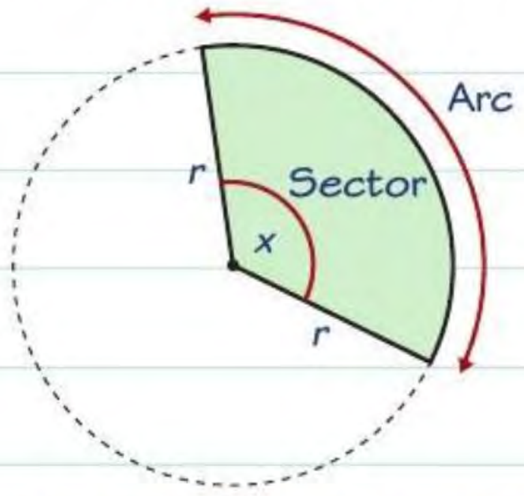
Tricky Topic

Sectors of circles

Each pair of radii divides a circle into two sectors, a **major sector** and a **minor sector**.



You can find the area of a sector by working out what fraction it is of the whole circle.



For a sector with angle x of a circle with radius r:

$$\text{Sector} = \frac{x}{360^\circ} \text{ of the whole circle so}$$

$$\text{Area of sector} = \frac{x}{360^\circ} \times \pi r^2$$

$$\text{Arc length} = \frac{x}{360^\circ} \times 2\pi r$$

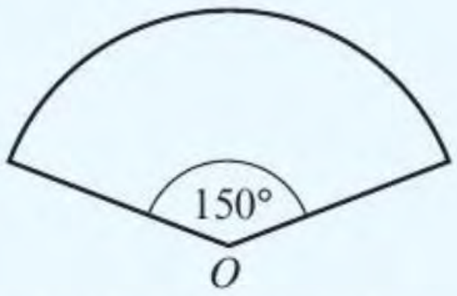
LEARN IT!

You can give answers in terms of π.
There is more about this on the next page.

Worked example

Target grade 5

The diagram shows a minor sector of a circle of radius 13 cm.



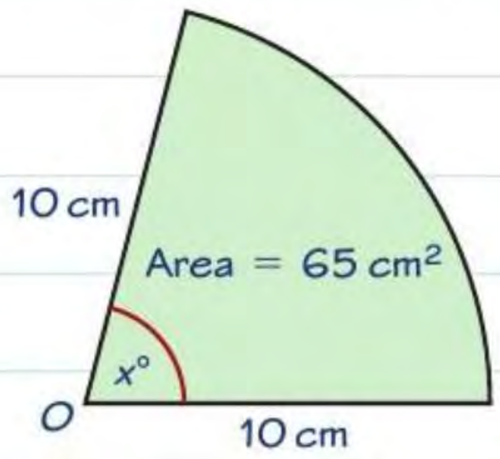
Work out the perimeter of the sector. **(4 marks)**

$$\text{Arc length} = \frac{x}{360^\circ} \times 2\pi r$$
$$= \frac{150^\circ}{360^\circ} \times 2 \times \pi \times 13$$
$$= 34.03392...$$
$$\text{Perimeter} = \text{Arc length} + \text{Radius} + \text{Radius}$$
$$= 34.03392... + 13 + 13$$
$$= 60 \text{ cm (2 s.f.)}$$

Don't round until your final answer. The radius is given correct to 2 significant figures so this is a good degree of accuracy.

Finding a missing angle

You can use the formulae for arc length or area to find a missing angle in a sector. Practise this method to help you tackle the hardest questions.

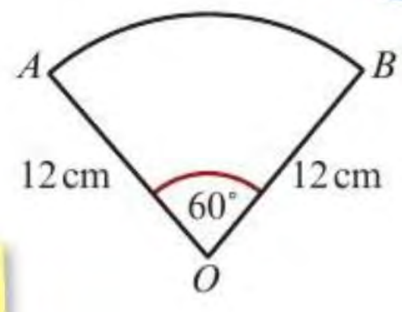


$$\text{Area of sector} = \frac{x}{360} \times \pi r^2$$
$$65 = \frac{x}{360} \times 100\pi$$
$$x = \frac{65 \times 360}{100\pi}$$
$$= 74.4845...$$
$$= 74.5^\circ \text{ (to 3 s.f.)}$$

Now try this

Target grade 5

OAB is a sector of a circle, centre O.
Angle $AOB = 60^\circ$.
$OA = OB = 12$ cm.
Work out the length of the arc AB.
Give your answer correct to 3 significant figures. **(3 marks)**



You need to learn the formula for arc length.

Had a look ☐ Nearly there ☐ Nailed it! ☐

Cylinders

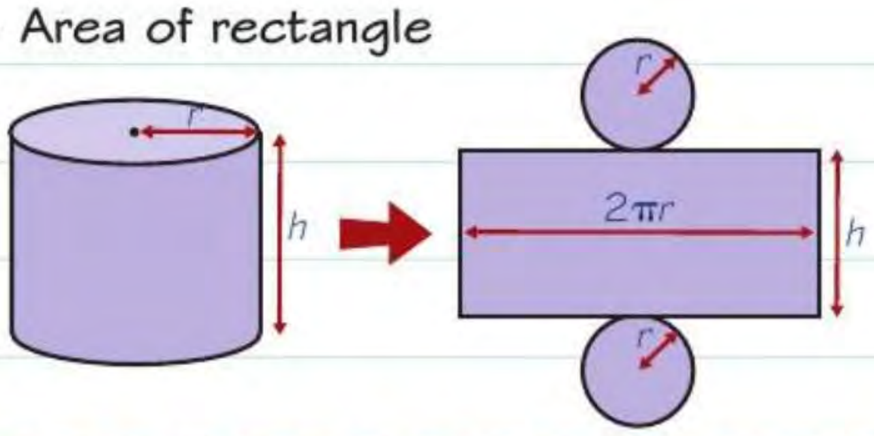
Surface area

To find the **surface area** of a **cylinder** you need to add up the areas of the faces.
A cylinder has two flat circular faces and one curved face. When you flatten out the curved face it is shaped like a rectangle.

Surface area = 2 × Area of circle + Area of rectangle

$= 2 \times \pi r^2 + 2\pi r \times h$

$= 2\pi r^2 + 2\pi rh$



The length of the rectangular face is the same as the circumference of the circle.

LEARN IT!

Volume

For a cylinder with radius r and height h:

Volume of cylinder = Area of base × Height

= Area of circle × Height

$= \pi r^2 h$

LEARN IT!

Problem solved! You should always write down the formula **before** substituting. Be careful when deciding which quantities to use. You are given the **diameter** but the formula uses **radius** so you need to divide by 2.

Worked example

Target grade 4

This tin of soup is in the shape of a cylinder with height 11 cm. The diameter of the base is 7 cm.

$1\text{ cm}^3 = 1\text{ ml}$

Work out the capacity of the tin in ml. **(3 marks)**

Radius of base = 7 ÷ 2 = 3.5 cm

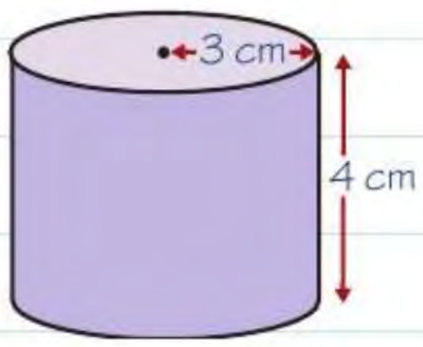
Volume = $\pi r^2 h = \pi \times 3.5^2 \times 11$

$= 423.3296...\text{ cm}^3$

The capacity is 423 ml to the nearest whole number.

In terms of π

If a question asks for an **exact value** or an answer **in terms of** π then don't use the π button on your calculator. Write your answer as a whole number or fraction multiplied by π.



Volume of cylinder = $\pi r^2 h$

$= \pi \times 3^2 \times 4$

Exact answer

Volume = $36\pi\text{ cm}^3$

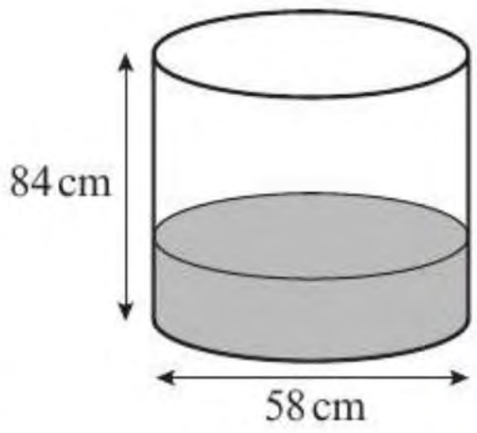
Rounded answer

Volume = 113 cm³ (to 3 s.f.)

Now try this

Target grade 4

The diagram shows an oil drum in the shape of a cylinder of height 84 cm and diameter 58 cm.
It is one-quarter full of crude oil.
Calculate the volume of oil in the cylinder.
Give your answer in litres, correct to the nearest litre. **(4 marks)**



1 litre = 1000 cm³

Had a look ☐ Nearly there ☐ Nailed it! ☐

Volumes of 3-D shapes

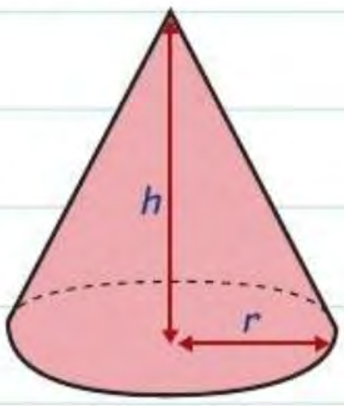
If you need to use these formulae in your exam they will be given to you with the question.

Cone

Volume of cone

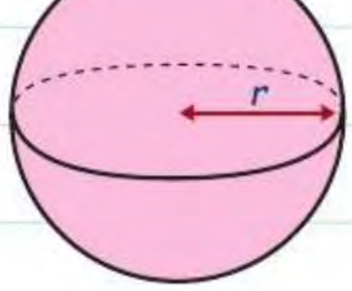
$= \frac{1}{3} \times$ Area of base $\times$ Vertical height

$= \frac{1}{3}\pi r^2 h$



Sphere

Volume of sphere $= \frac{4}{3}\pi r^3$

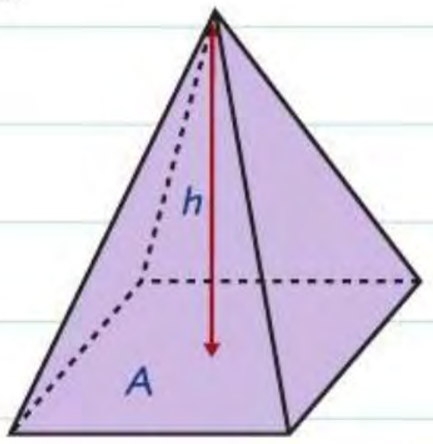


Pyramid

Volume of pyramid

$= \frac{1}{3} \times$ Area of base $\times$ Vertical height

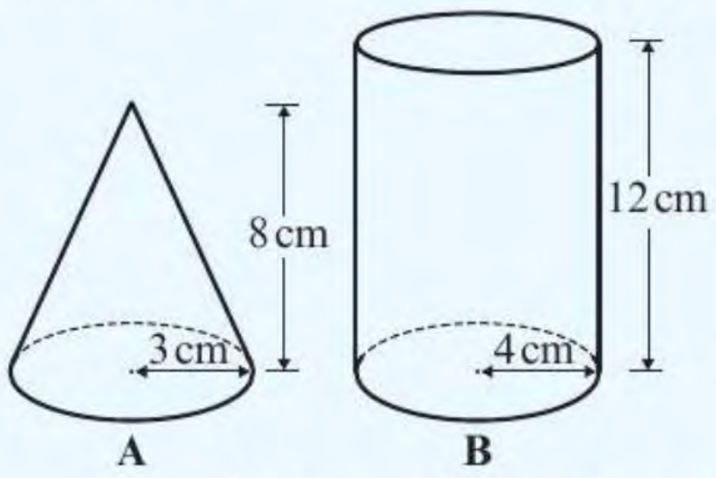
$= \frac{1}{3}Ah$



Worked example

Target grade 5

The diagram shows a cone A and a cylinder B. Show that the volume of B is 8 times the volume of A. **(4 marks)**



Volume of A $= \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times 3^2 \times 8$

$= 24\pi$

Volume of B $= \pi r^2 h = \pi \times 4^2 \times 12$

$= 192\pi$

$8 \times 24\pi = 192\pi$ so the volume of B is 8 times the volume of A.

Examiners' report

You might have to **compare** two volumes or areas in your exam. These questions might involve:

- working out the ratio between two different areas or volumes
- finding an unknown quantity represented by a letter
- finding an expression for a length, area or volume in terms of an unknown.

In this question you need to know the ratio between the two volumes. Calculate them both, then write a short **conclusion**. Make sure you show the calculation in your conclusion: $8 \times 24\pi = 192\pi$

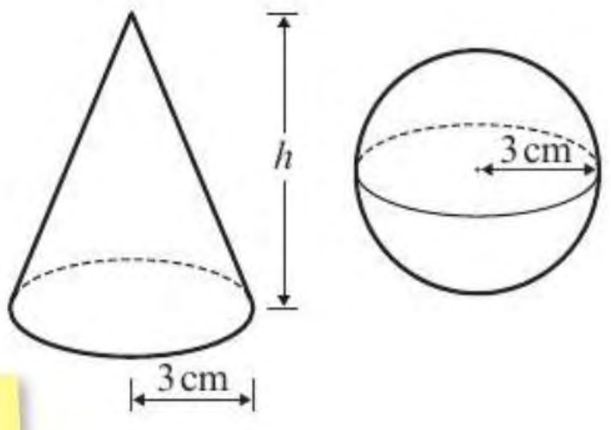
You can leave your working in terms of π to make it easier. There is more about this on page 106.

Real students have struggled with questions like this in recent exams – **be prepared!**

Now try this

Target grade 5

The radius of the base of a cone is 3 cm and its height is h cm.
The radius of a sphere is 3 cm.
The volume of the cone is equal to the volume of the sphere.
Find the value of h. You must show your working. **(3 marks)**



Write the expression for the volume of each shape; then set them equal to each other. Rearrange to make h the subject.

Had a look ☐ Nearly there ☐ Nailed it! ☐

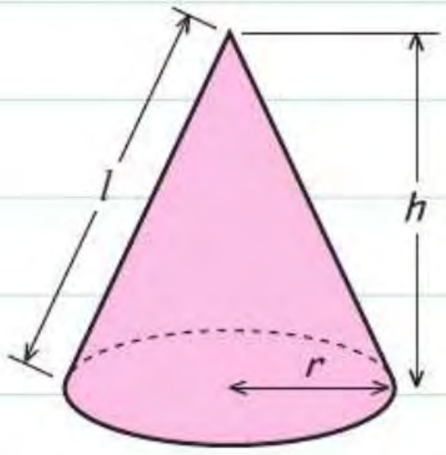
Tricky Topic

Surface area

Cone

The formula for the **curved surface area** of a cone will be given if you need it for a question.

Curved surface area of cone = πrl



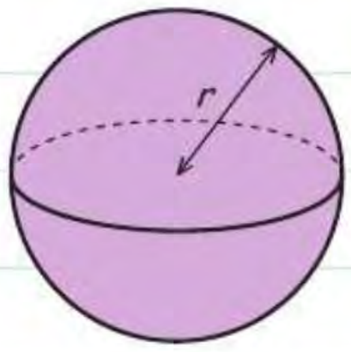
Be careful! This formula uses the slant height, l, of the cone.

To calculate the **total** surface area of the cone you need to add the area of the base. Surface area of cone = $\pi r^2 + \pi rl$

Sphere

The formula for the surface area of a sphere will be given if you need to use it.

Surface area of sphere = $4\pi r^2$



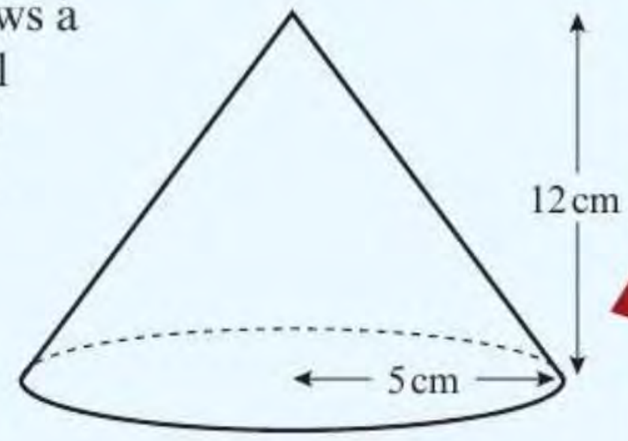
For a reminder about areas of circles and surface areas of cylinders have a look at pages 104 and 106.

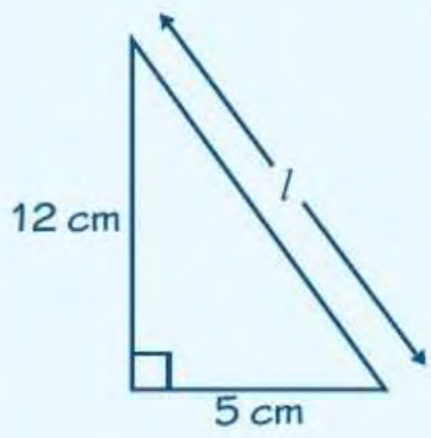
A hemisphere is half a sphere, so the area of the curved surface is $\frac{1}{2} \times 4\pi r^2$

Worked example

Target grade 5

The diagram shows a cone with vertical height 12 cm and base radius 5 cm. Work out the curved surface area of the cone. **(4 marks)**





$l^2 = 12^2 + 5^2 = 169$

$l = 13$ cm

Curved surface area

$= \pi rl$

$= \pi \times 13 \times 5$

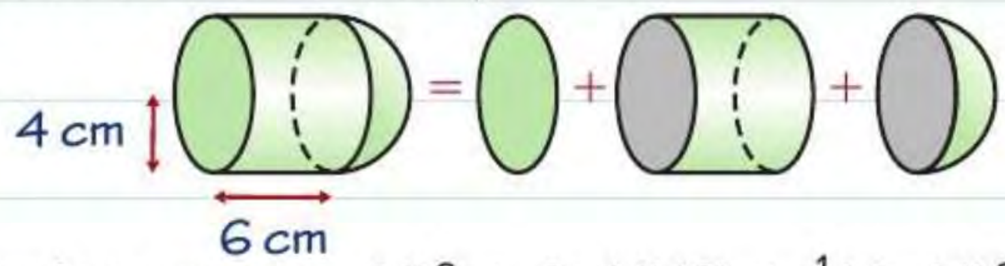
$= 65\pi$ cm^2

Problem solved! To work out the curved surface area you need to know the radius and the slant height. You are given the radius and the **vertical height**.

To calculate the slant height you need to use Pythagoras' theorem. Sketch the right-angled triangle containing the missing length.

Compound shapes

You can calculate the surface area of more complicated shapes by adding together the surface area of each part.



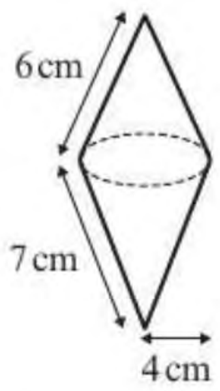
Surface area = $\pi(4)^2 + 2\pi(4)(6) + \frac{1}{2}[4\pi(4)^2]$

$= 96\pi$ cm^2

Now try this

Target grade 5

1 The diagram shows an object made from two cones, placed one on top of the other.
The top cone has a slant height of 6 cm and the bottom cone has a slant height of 7 cm.
Both cones have a radius of 4 cm.
Work out the total surface area of the object.
Give your answer in terms of π. **(4 marks)**



2 A solid object is formed by joining a hemisphere to a cylinder.
Both the hemisphere and the cylinder have a radius of 2.1 cm. The cylinder has a height of 5.6 cm. Work out the total surface area of the object. Give your answer to 3 significant figures. **(4 marks)**