# Gradients of lines

The **gradient** of a straight-line graph measures how steep the line is. You can work out the gradient by drawing a triangle and using this rule: $\text{Gradient} = \dfrac{\text{Distance up}}{\text{Distance across}}$

Worked example

Target grade 3

This scatter graph shows the relationship between the budgets of some films and the amounts of money they made at the box office in their opening weekends. A line of best fit has been drawn on the scatter graph.

Opening weekend (£ million) vs *Budget (£ million)*
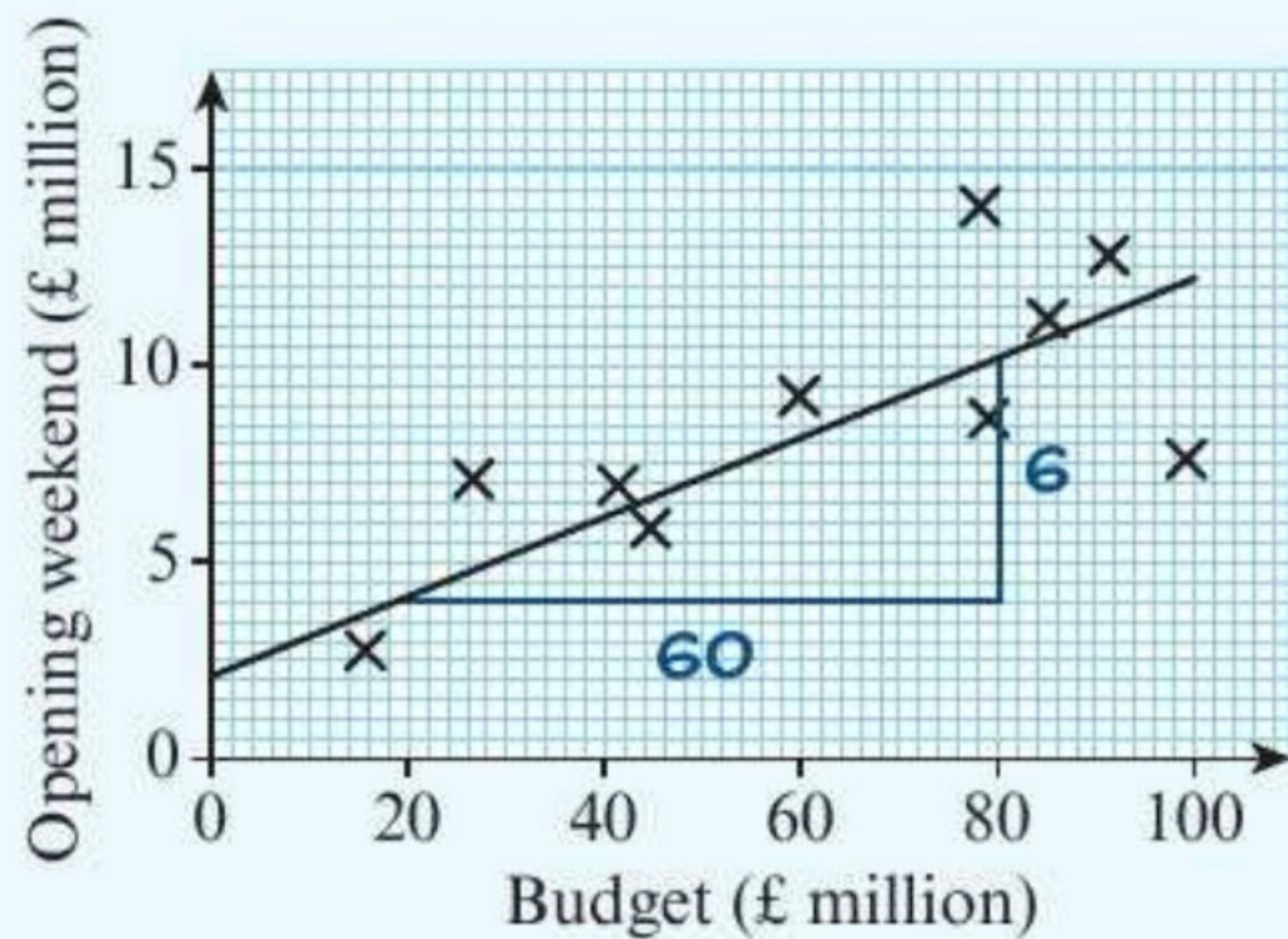
> **Everything in blue is part of the answer.**

> To work out the gradient of a line you need to draw a triangle.
>
> Write the distance across and the distance up.
>
> **Watch out** for the scales on the axes:
>
> Distance across = 80 – 20 = 60
>
> Distance up = 10 – 4 = 6
>
> There is more on scatter graphs on page 119.

(a) Work out the gradient of the line of best fit. **(1 mark)**

$$\text{Gradient} = \frac{\text{Distance up}}{\text{Distance across}} = \frac{6}{60} = 0.1$$

(b) Amir says, 'Spending an extra £10 million on a film is likely to increase the amount of money taken during its opening weekend by £1 million.' Does the graph support this statement? **(1 mark)**

Yes. The gradient of the line of best fit is 0.1, so the amount of money taken during the opening weekend increases by approximately £0.1 million for each additional £1 million of budget.

Top triangle tips!

1. Draw one side of your triangle on a large grid line as you are less likely to make a mistake in your calculations.

2. Use a large triangle as this means your calculations are more accurate.

3. Don't just count grid squares. Use the scale to work out the distance across and the distance up.

Positive or negative?

Positive gradient

Gradient = 0
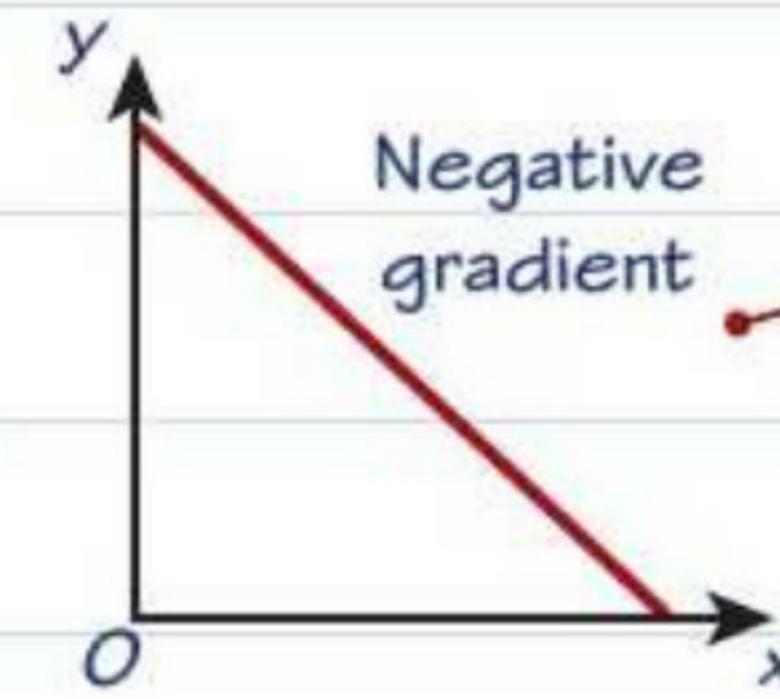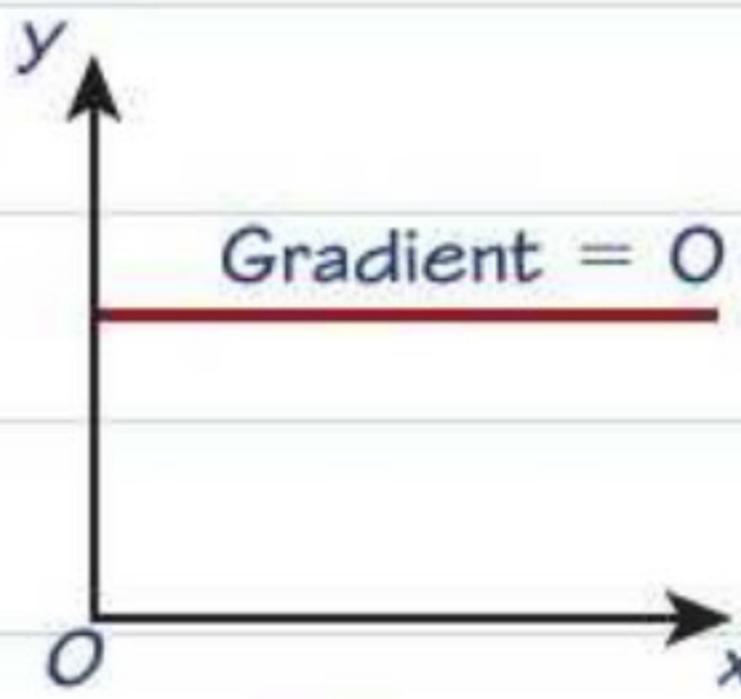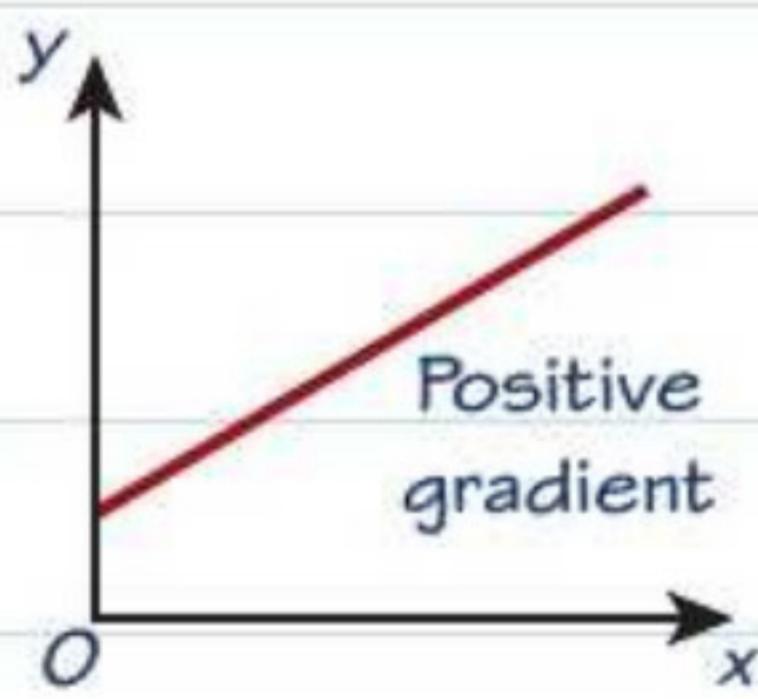
Negative gradient

> If the gradient is negative then one value decreases as the other value increases.

Now try this

Target grade 3

> Use the **scale** when calculating the gradient.

The scatter graph shows a relationship between the amount of money a company spent on marketing each month and its revenue.

(a) Calculate the gradient of the line of best fit.

(b) Amy states that 'For every £10 000 spent on advertising, revenue increases by £20 000.'
Does the scatter graph support her statement? **(1 mark)**

Revenue (£1000s) vs *Money spent (£1000s)*

Hot Topic
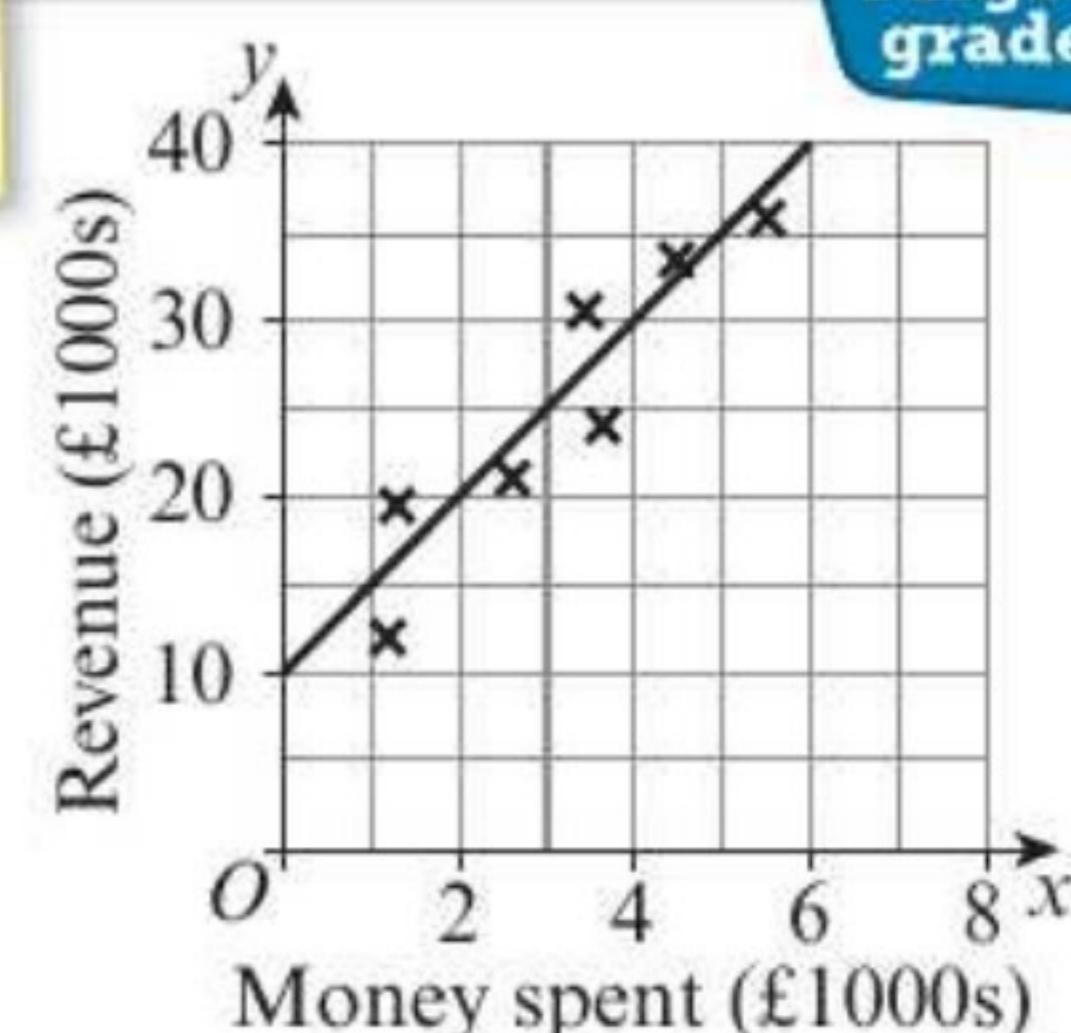
Straight-line graphs 1

Here are two things you need to know about straight-line graphs:

1 If an equation is in the form $y = mx + c$, its graph will be a straight line.

$$y = -\tfrac{1}{2}x + 5$$

This number tells you the gradient of the graph.
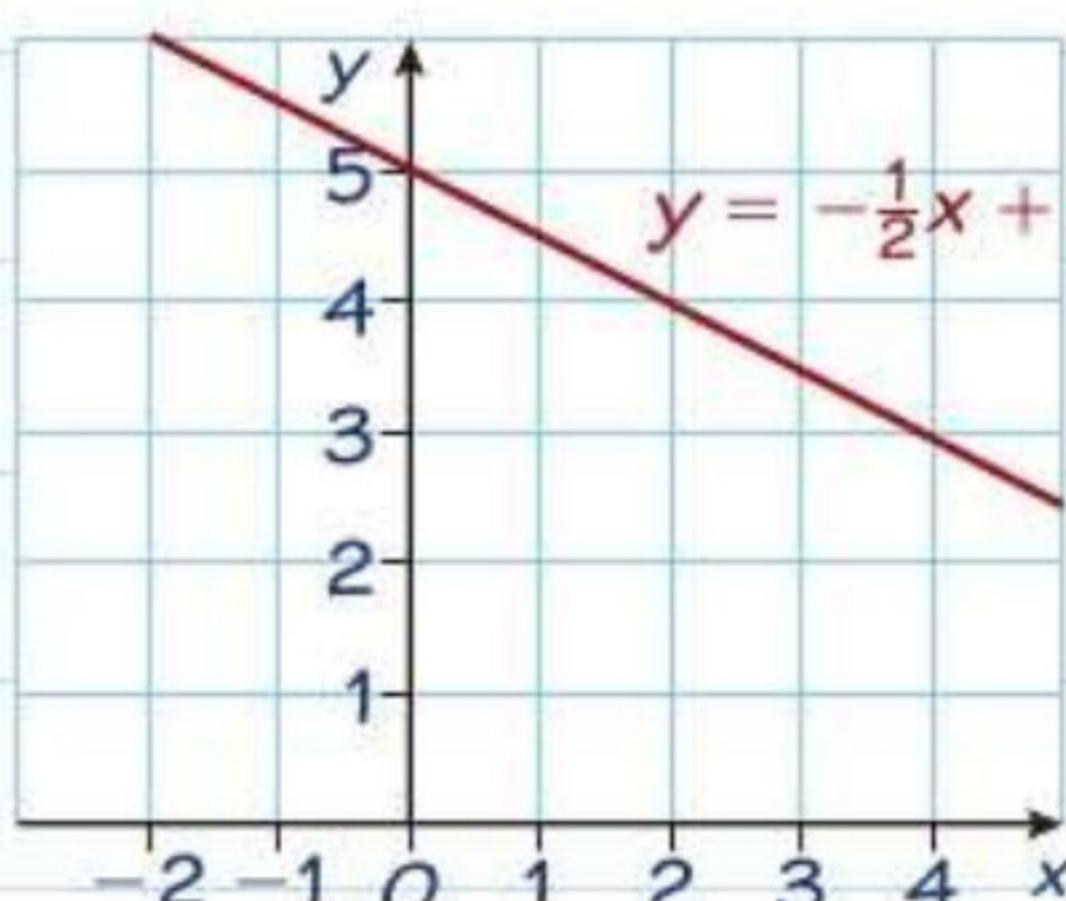
The y-intercept of the graph is at (0, 5).

$y = -\tfrac{1}{2}x + 5$

The gradient of the graph is $-\tfrac{1}{2}$. This means that for every unit you go across, you go half a unit down.

2 Use a table of values to draw a graph.

$y = 2x + 1$

| x	−1	0	1	2
y	−1	1	3	5

$y = 2 \times 2 + 1 = 5$

$y = 2x + 1$
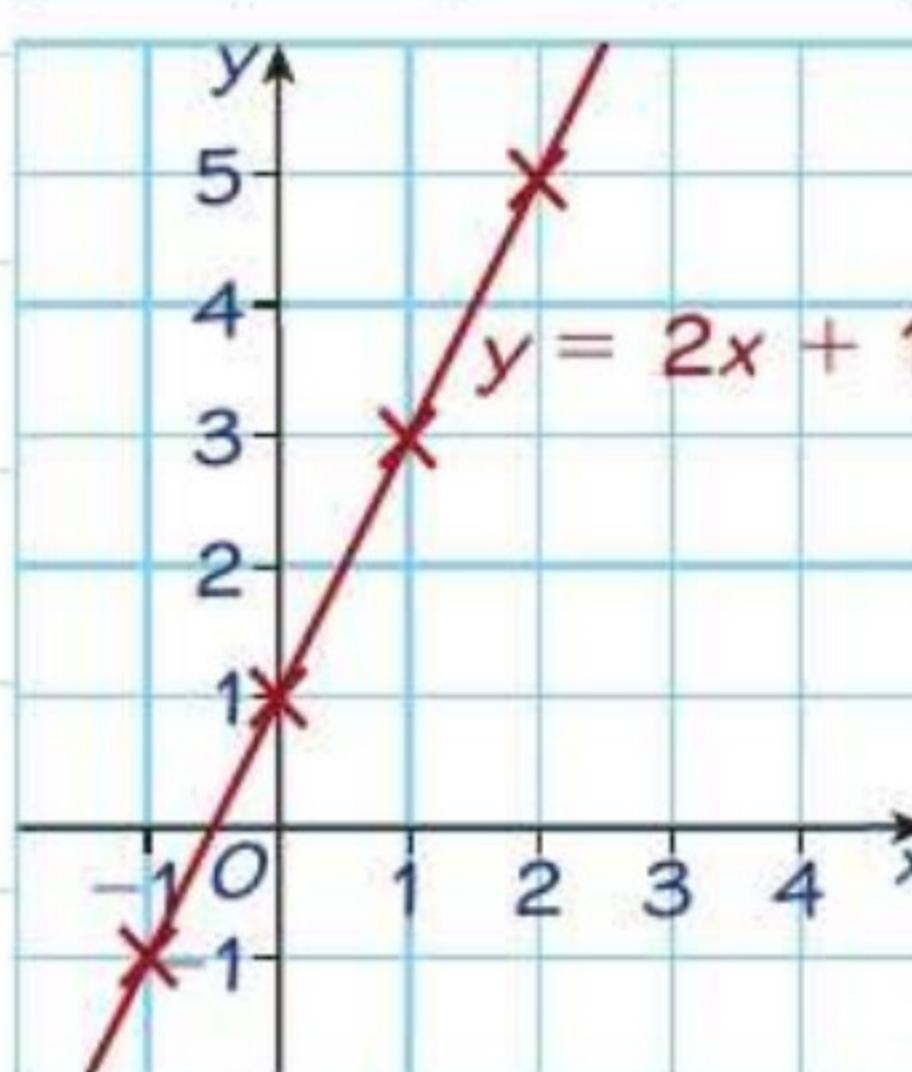
Choose simple values of x and substitute them into the equation to find the values of y.

Plot the points on your graph and join them with a straight line.

Worked example

Target grade 4

On the grid draw the graph of $x + y = 4$ for values of x from −2 to 5.

(3 marks)

$x + y = 4$
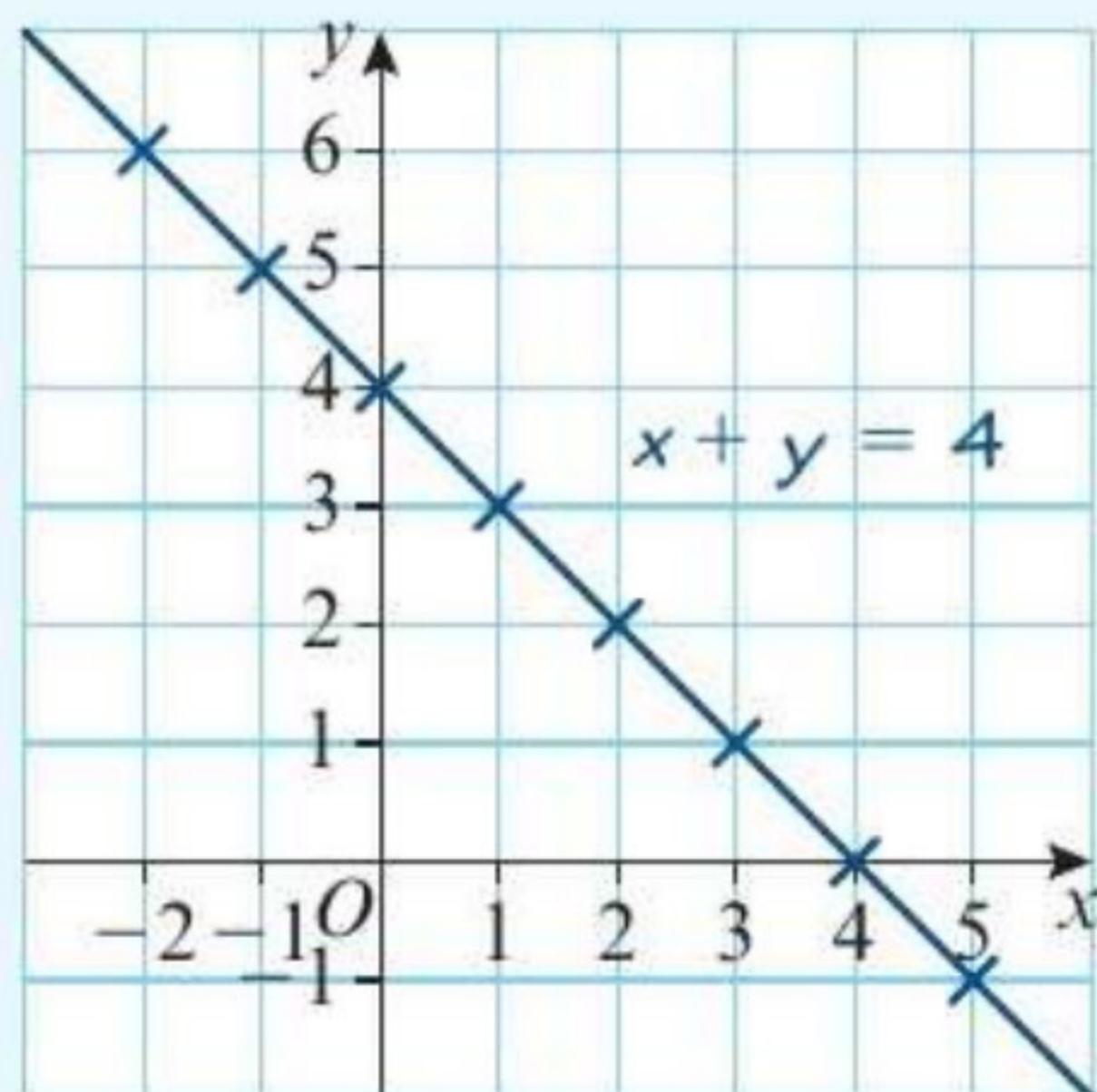
Everything in blue is part of the answer.

x	−2	−1	0	1	2	3	4	5
y	6	5	4	3	2	1	0	−1

Finding equations

If you have a graph you can find its equation by working out the gradient and looking at the y-intercept.
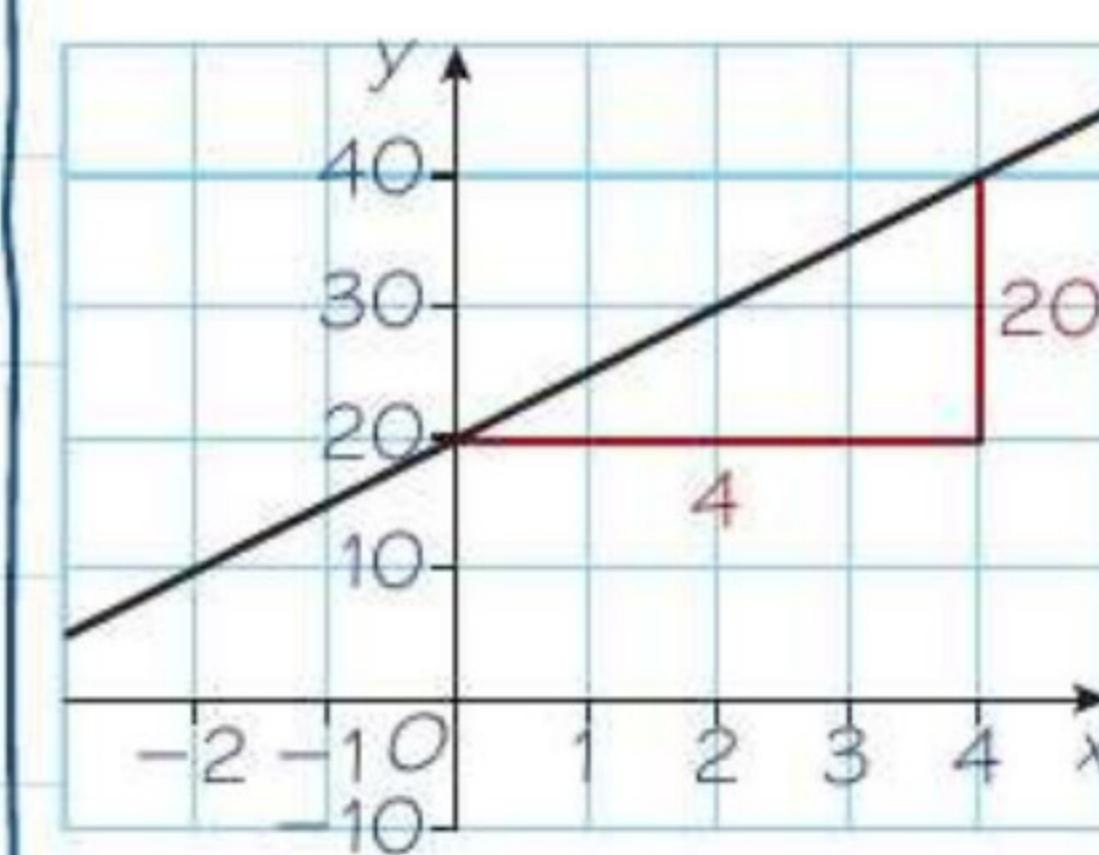
Draw a triangle to find the gradient.

Gradient $= \dfrac{20}{4} = 5$

The y-intercept is (0, 20).

Put your values for gradient, m, and y-intercept, c, into the equation of a straight line, $y = mx + c$.

The equation is $y = 5x + 20$

You can rearrange the equation of this graph into the form $y = mx + c$ so it is a **straight line**.

$x + y = 4 \qquad (-x)$

$\quad y = -x + 4 \qquad m = -1$ and $c = 4$

The gradient is −1 and the y-intercept is at (0, 4). You could use this information to draw the graph, but it's safer to make a table of values. Make sure you plot **at least three** points, then join them with a straight line **using a ruler**.

Now try this

Target grade 4

Find the equation of the straight line.

(3 marks)
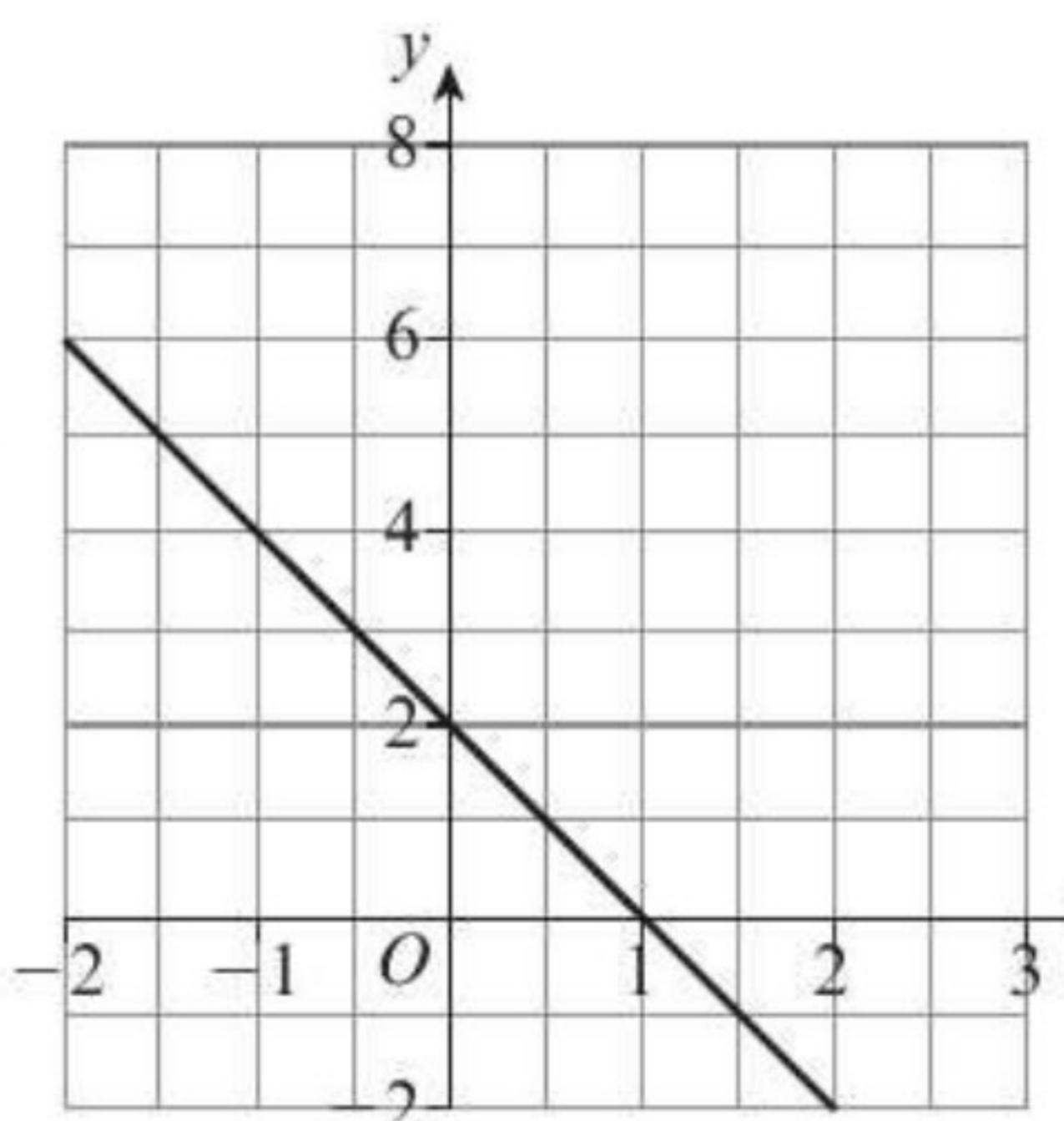
Use $y = mx + c$. Draw a triangle to find the gradient, m. The graph slopes down so m will be **negative**.

Straight-line graphs 2

You can use **algebra** to find the equation of the line when you are given one point and the gradient, or two points.

① Given one point and the gradient

| Substitute the gradient for m in $y = mx + c$ |
| Substitute the x- and y-values given into the equation |
| Solve the equation to find c |
| Write out the equation |

Gradient 2, passing through point (3, 7)

$y = 2x + c$
$7 = 2 \times 3 + c$
$7 = 6 + c$ (-6)
$c = 1$

$y = 2x + 1$

② Given two points

| Draw a sketch showing the two points |
| Work out the gradient of the line with a triangle |
| Use method 1 (on the left) and one of the points given to find the equation |

Passing through points (8, 20) and (10, 30)

(10, 30)
10
(8, 20)
2
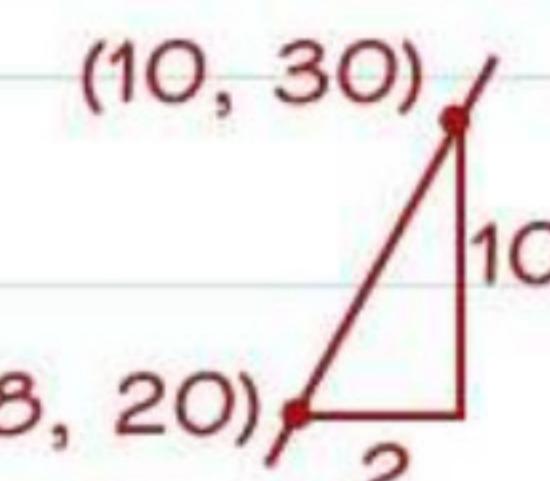
Gradient $= \dfrac{10}{2} = 5$

$y = 5x + c$
$30 = 5 \times 10 + c$
so $c = -20$

$y = 5x - 20$

Parallel lines

Parallel lines have the same gradient.
These three lines all have a gradient of 1.

$y = x$
$y = x + 2$
$y = x - 3$
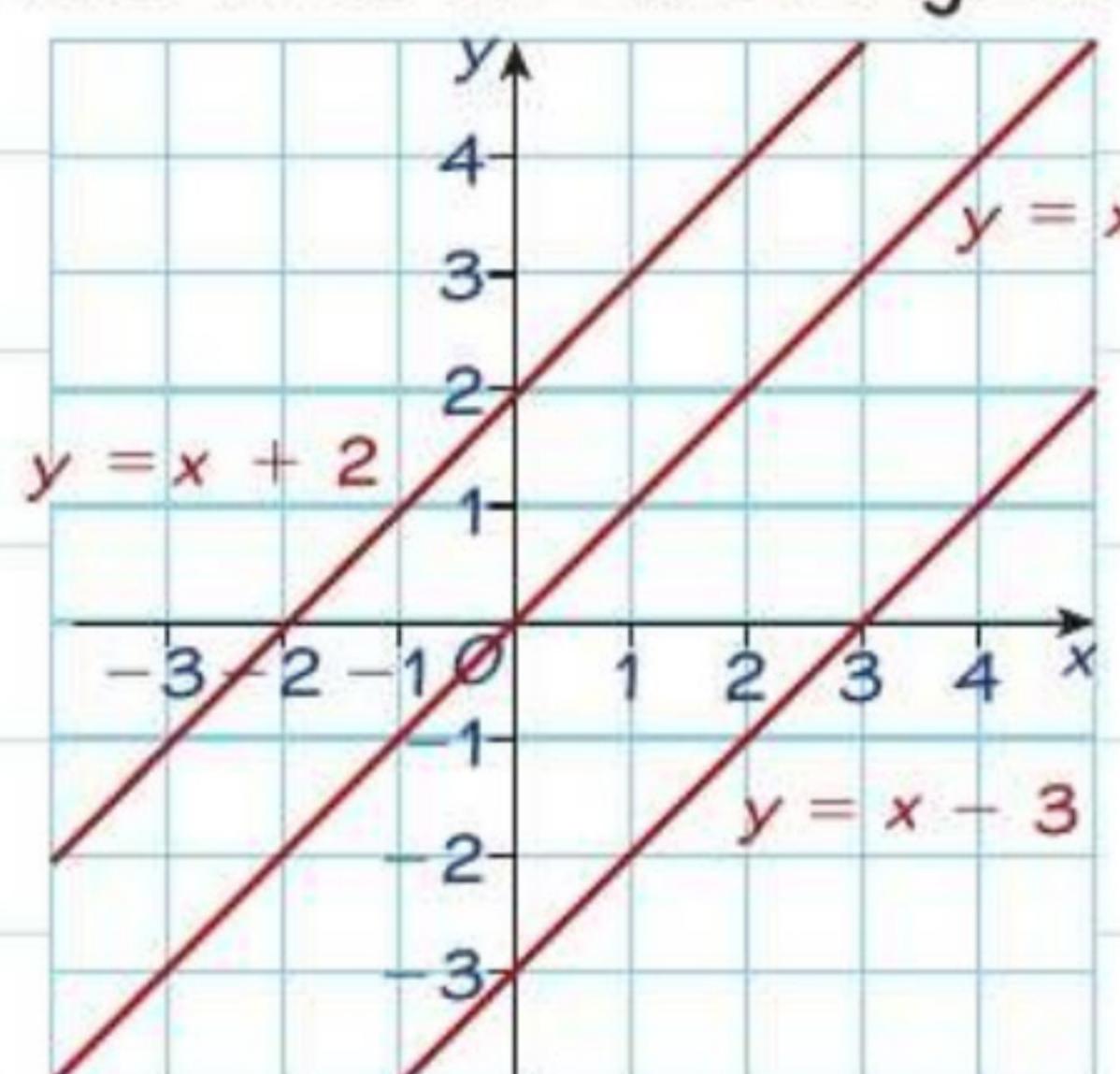
Parallel lines **never meet**.

Worked example

Target grade 5

Line A has equation $y = \frac{1}{2}x + 1$
Line B is parallel to line A.

Line B — P
$y = \frac{1}{2}x + 1$
Q
Line A
O   6 x
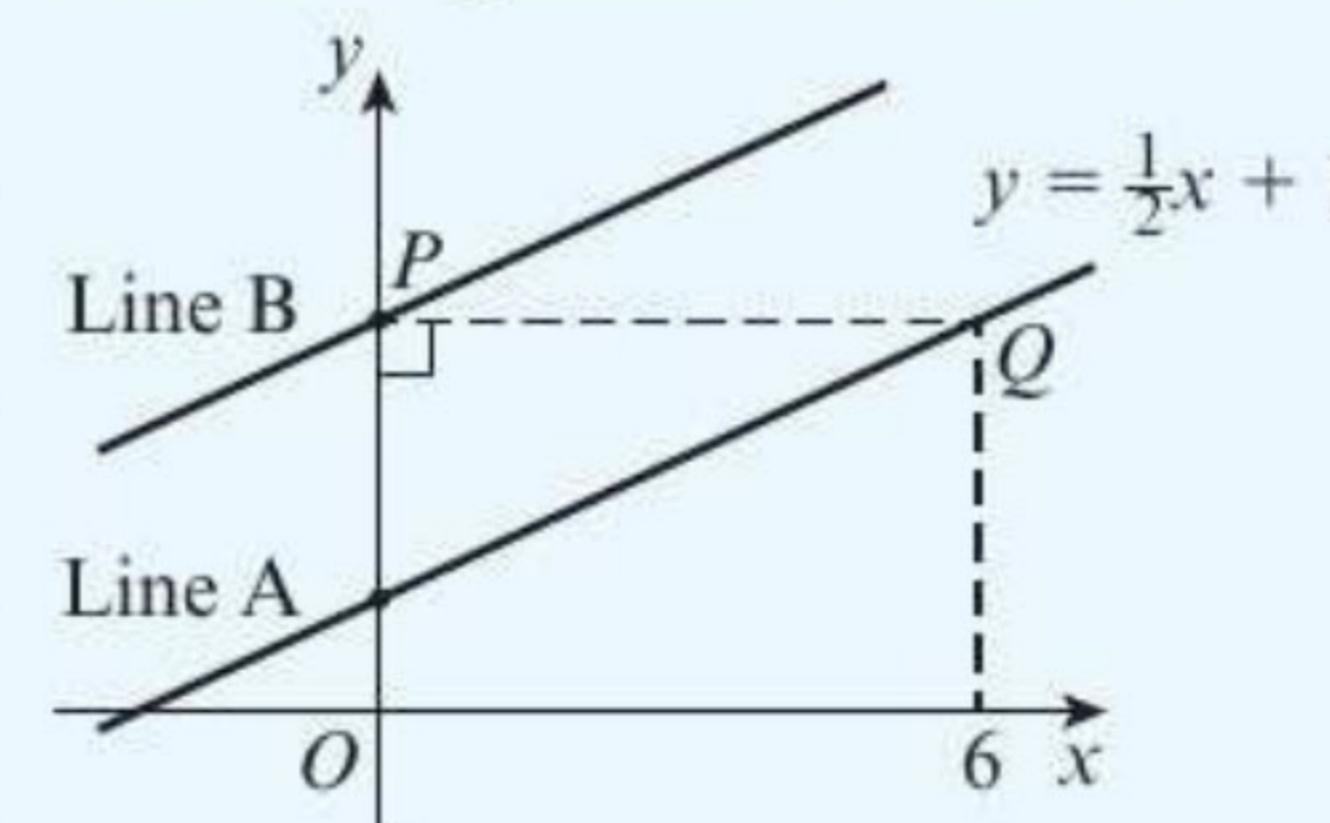
Work out the equation of line B.   (3 marks)
$y = \frac{1}{2}x + 1$
$\quad = \frac{1}{2} \times 6 + 1 = 3 + 1 = 4$

So Q has coordinates (6, 4)
So P has coordinates (0, 4)
Line B is parallel to line A so also has
gradient $\frac{1}{2}$
So equation of line B is $y = \frac{1}{2}x + 4$
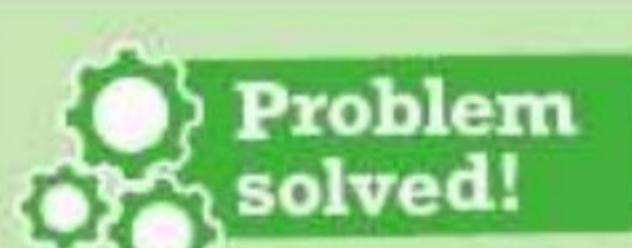
Problem solved! Plan your strategy before you start:

1. Use the equation of line A to find the coordinates of points P and Q.
2. Use the fact that line A is parallel to line B to write down the gradient of line B.
3. Use $y = mx + c$ to write down the equation of line B.

Now try this

Worked solution video

Target grade 5

1 A straight line has gradient 6 and passes through the point (5, 10). Find the equation of the line.   (2 marks)
2 Find the equation of the straight line which passes through the points (3, 5) and (7, 13).   (3 marks)

Find the gradient, m, then substitute $x = 3$ and $y = 5$ into $y = mx + c$. Solve the equation to find the value of c.