Had a look ☐ Nearly there ☐ Nailed it! ☐

Tricky Topic

Factorising quadratics

You need to be able to **factorise** quadratic expressions by writing them as the product of **two brackets**. Make sure you are really confident expanding **double brackets** before you start. You can revise this on page 43. You can also revise factorising simpler expressions on page 29.

Expanding brackets

$(x + 2)(x + 3) = x^2 + 5x + 6$

Factorising

Factorising $x^2 + bx + c$

You need to write the expression with **two brackets**. Look for two numbers whose **sum** is equal to b and whose **product** is equal to c.

You need to find two numbers which add up to 7... $5 + 2 = 7$

$x^2 + 7x + 10 = (x + 5)(x + 2)$

... and multiply to make 10 $5 \times 2 = 10$

You can use this table to help you find the two numbers.

b	c	Factors
positive	positive	both numbers positive
positive	negative	bigger number positive and smaller number negative
negative	negative	bigger number negative and smaller number positive
negative	positive	both numbers negative

Worked example

Target grade 5

Factorise $x^2 - x - 20$ **(2 marks)**

Factor pairs of 20:
1 and 20, 2 and 10, 4 and 5

$x^2 - x - 20 = (x + 4)(x - 5)$

Check:

$(x + 4)(x - 5) = x^2 - 5x + 4x - 20$
$= x^2 - x - 20$ ✓

Examiners' report

The answer will have **two sets of brackets**.
The last term is negative, so the brackets will have one + sign and one − sign: $(x + \square)(x - \square)$.
The numbers will be a **factor pair of 20**.
With any factorisation, the safest thing to do is to **check your answer** by expanding the brackets.

Real students have struggled with questions like this in recent exams – **be prepared!**

Difference of two squares

You can factorise expressions that are written as

(something)^2 − (something else)^2

Use this rule:

$a^2 - b^2 = (a + b)(a - b)$

LEARN IT!

$x^2 - 36 = x^2 - 6^2$
$= (x + 6)(x - 6)$

36 is a square number.
$36 = 6^2$ so $a = x$ and $b = 6$

Worked example

Target grade 5

Factorise $x^2 - 9$ **(2 marks)**

$x^2 - 9 = (x + 3)(x - 3)$

Check:

$(x + 3)(x - 3) = x^2 - 3x + 3x - 9$
$= x^2 - 9$ ✓

Keep an eye out for the difference of two squares. $9 = 3^2$ so $a = x$ and $b = 3$.

Now try this

Target grade 5

Worked solution video

1 Factorise
(a) $x^2 + 6x + 8$ **(2 marks)**
(b) $x^2 - 10x + 16$ **(2 marks)**

2 Factorise
(a) $x^2 - 144$ **(2 marks)**
(b) $x^2 - 49$ **(2 marks)**

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Tricky Topic

Quadratic equations

You need to be able to **solve** a quadratic equation **without a calculator** by factorising one side. If you need to solve a quadratic equation in your Foundation GCSE exam it will look something like this:

The left-hand side will be a **quadratic expression.**

$$x^2 + 6x + 5 = 0$$

The right-hand side will be **zero.**

Follow the steps below to solve the equation.

1 Factorise the left-hand side of the equation:
$(x + 5)(x + 1) = 0$

For a reminder about factorising quadratic expressions look at page 46.

2 Set each factor equal to zero:
$(x + 5)(x + 1) = 0$
$(x + 5) = 0$ $(x + 1) = 0$

3 Solve the two linear equations to find the two solutions:
$x + 5 = 0$ $x + 1 = 0$
$x = -5$ $x = -1$

Write both solutions in the form $x =$ ☐

Worked example

Target grade 5

Solve $x^2 + 8x - 9 = 0$ **(3 marks)**

$(x + 9)(x - 1) = 0$

$x + 9 = 0$ $x - 1 = 0$

$x = -9$ $x = 1$

1. Start by factorising the left-hand side of the equation. You can check by expanding the brackets:
   $(x + 9)(x - 1) = x^2 - x + 9x - 9$
   $= x^2 + 8x - 9$ ✓
2. Set each factor equal to zero.
3. Solve the linear equations to get **two** solutions.

Check it!
$(1)^2 + 8(1) - 9 = 1 + 8 - 9 = 0$ ✓
$(-9)^2 + 8(-9) - 9 = 81 - 72 - 9 = 0$ ✓

Worked example

Target grade 5

Solve $x^2 - 100 = 0$ **(3 marks)**

$(x + 10)(x - 10) = 0$

$x + 10 = 0$ $x - 10 = 0$

$x = -10$ $x = 10$

You can also use inverse operations:
$x^2 - 100 = 0$ $(+ 100)$
$x^2 = 100$ $(\sqrt{\ })$
$x = 10$ or $x = -10$
There are two answers because $(-10)^2 = -10 \times -10 = 100$

Two to watch

1 When one solution is $x = 0$
$x^2 - 10x = 0$
$x(x - 10) = 0$
Solutions are $x = 0$ and $x = 10$

2 Difference of two squares
$x^2 - 25 = 0$
$(x - 5)(x + 5) = 0$
Solutions are $x = 5$ and $x = -5$

Now try this

Target grade 5

For parts (a) and (c) look at the blue box above.

1 (a) Factorise $x^2 + 5x + 6$ **(2 marks)**
(b) Write down the two solutions of $x^2 + 5x + 6 = 0$ **(1 mark)**

Both solutions are **negative.**

2 Solve
(a) $x^2 - 5x = 0$ **(2 marks)**
(b) $x^2 + 3x - 28 = 0$ **(3 marks)**
(c) $x^2 - 144 = 0$ **(3 marks)**

Had a look ☐ Nearly there ☐ Nailed it! ☐

Using quadratic graphs

You might need to read values off a quadratic graph. You can do this by looking at the points where the graph crosses the x-axis, or by drawing a horizontal line on your graph.

Worked example

Target grade 4

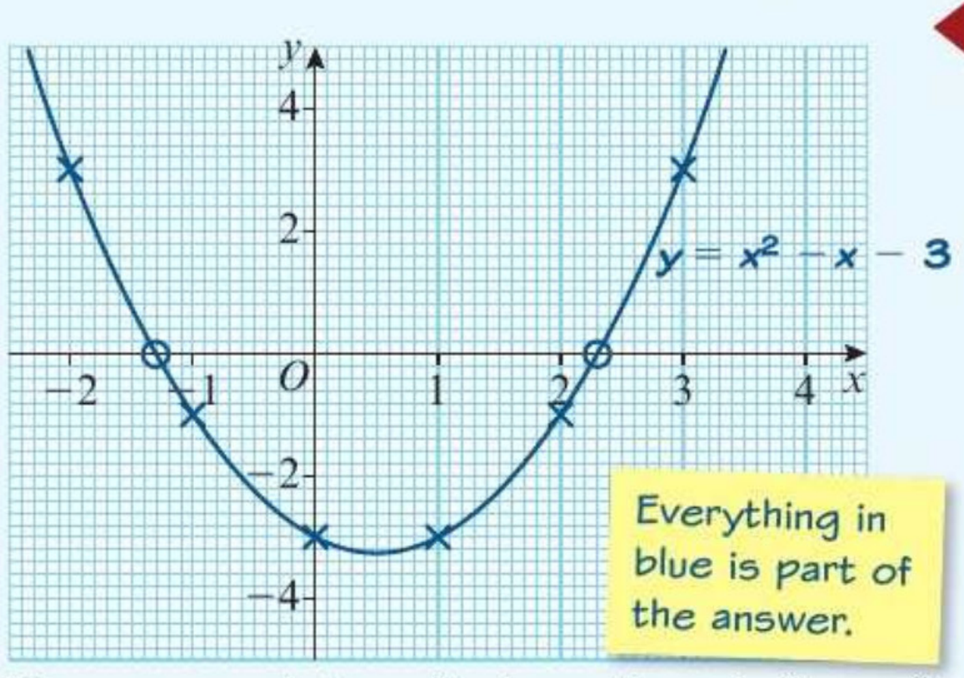
(a) Complete the table of values for $y = x^2 - x - 3$

(2 marks)

x	−2	−1	0	1	2	3
y	3	−1	−3	−3	−1	3

(b) On the grid, draw the graph of $y = x^2 - x - 3$

(2 marks)



Everything in blue is part of the answer.

(c) Use your graph to write down the solutions of $x^2 - x - 3 = 0$

(2 marks)

$x = -1.3$ and $x = 2.3$

This is an example of a **quadratic graph**. Its equation contains an x^2 term. You should always use a table of values to draw quadratic graphs.

Examiners' report

Don't use a ruler – you need to join your points with a **smooth curve**.

Real students have struggled with questions like this in recent exams – **be prepared!**

The solutions of the equation $x^2 - x - 3 = 0$ are the x-coordinates at the points where the graph crosses the x-axis. These are sometimes called the **roots** of the equation. You should always read graphs accurate to the nearest small square. So on this graph your answers will be accurate to 1 decimal place.

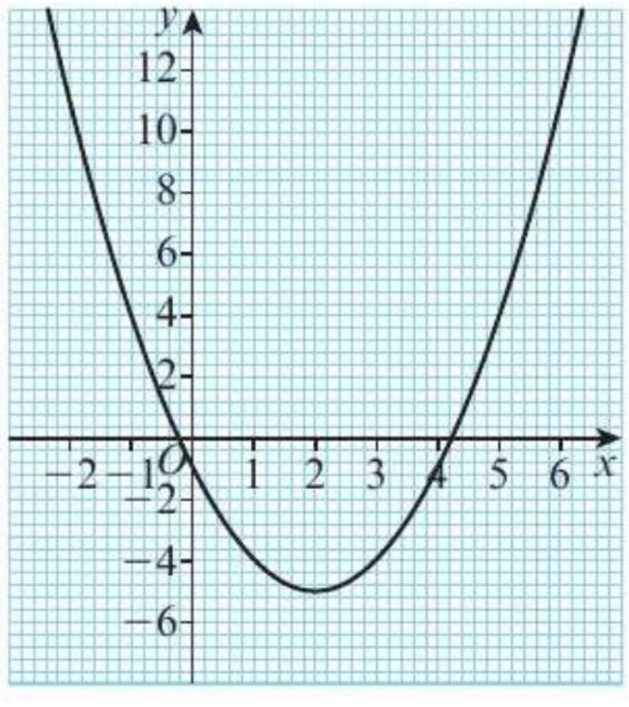
Now try this

Target grade 4

This is the graph of $y = x^2 - 4x - 1$

(a) Write down the values of x where the graph crosses the x-axis. **(2 marks)**

(b) Write down the values of x when $y = 8$ **(2 marks)**



Draw a horizontal line on the graph at $y = 8$.

Read down to the x-axis at the points where the line crosses the curve.

Read the graph correct to the nearest small square.

These values of x are the solutions to the quadratic equation $x^2 - 4x - 1 = 8$